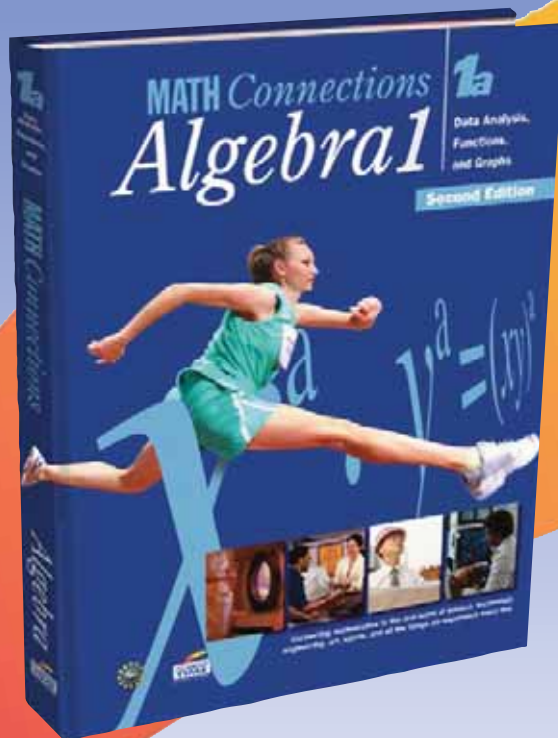


# MATH *Connections*<sup>®</sup>

## *Second Edition 2009*

Designed to be implemented as a stand-alone Algebra 1, Geometry, and Algebra 2 core curriculum, or as an integrated approach to meet the challenges of the 21st century.



### **Year 1: Books, 1a and 1b**

#### **Algebra 1:**

**Data Analysis, Functions, and Graphs**

### **Year 2: Books, 2a and 2b**

#### **Geometry:**

**Geometric Concepts in Two and Three Dimensions**

### **Year 3: Books, 3a and 3b**

#### **Algebra 2:**

**Functions and Modeling**



# Finally, a curriculum that works for all your students, even the students that have been struggling with mathematics.

## Skills, Problem-solving, Conceptual Thinking, and Connections

Without the above four basic elements, a mathematics curriculum just won't stand up to the needs of all your students — *especially your students who have been struggling with math*. Two, out of four simply won't help all your students fully comprehend math for life-long learning.

That's precisely what sets MATH *Connections*® apart from traditional mathematics curricula.

MATH *Connections* focuses not only on skills and problem-solving found in other secondary math programs, but it also focuses on conceptual thinking and the connections that make mathematical sense to students.

## It begins with the right connections.

A standards-based secondary curriculum MATH *Connections*, as its name suggests, is built around connections between mathematics and the real world of people, business, and everyday life.

- *Connections* between mathematics and science.



- *Connections* between mathematics and other areas such as history, literature, and art.



- *Connections* between different mathematical areas.

And, it makes these connections meaningful for students using algebra, geometry, probability, statistics, trigonometry, and discrete mathematics.

**MATH *Connections* implements STEM and work-force-readiness-21st-century skills: technological literacy, critical thinking, analytical ability, building on the works of others, and working collaboratively.**

MATH *Connections* encourages students to understand math concepts, reason things out, ask their own questions, and look for patterns — rather than just calculating of numbers.

You will find that your students will gain a fuller, richer understanding of mathematics. And, they will be better able to apply what they've learned throughout their lives, no matter what profession they choose in the 21st century.

# MATHConnections® Algebra 1

**Year 1: Book 1a** starts with data presentation and analysis, then presents the algebra of first-degree equations and the coordinate geometry of straight lines so that students can use the linear regression capabilities of a graphing calculator to do defensible forecasting in real-world settings. **Year 1: Book 1b** generalizes and expands these ideas to techniques for solving two linear equations in two unknowns and interpreting such solutions in real-world contexts, then to functions in general, which are linked with the algebra and coordinate geometry already developed. The function concept is extended to basic combinatorics and probability. This year ends with a detailed look at quadratic functions and equations, using numerical, graphical, and algebraic methods.

## Chapter 1: Turning Facts into Ideas

- 1.1 **Data Types of data, data collection, bar graphs, pie charts**
- 1.2 **Finding the Mean** Dotplots, mean, mode, frequency
- 1.3 **Displaying Data by Grouping** Using a graphing calculator to manipulate data, histograms, stem-and-leaf plots
- 1.4 **Another Center: The Median** Median the effect of outliers, appropriate measures of center
- 1.5 **Boxplots** Five-number summaries, boxplots, comparison of data sets
- 1.6 **Absolute Value and Deviation**, Absolute value, distance on a number line, the spread of a data set, mean absolute deviation
- 1.7 **Using Squares and Square Roots** Square roots, variance, standard deviation

### Looking Back, Review Exercises

## Chapter 2: Welcome to Algebra

- 2.1 **Abbreviations All Around Us** Abbreviations and acronyms, algebraic symbols for quantities and relationships, unknown values, and algebraic formulas
- 2.2 **Algebra Is Abbreviations** Variables, constants, formulas and equations that describe real-world relationships
- 2.3 **Some Laws of Algebra** Commutative, associative, and distributive laws for addition and multiplication, order of operations
- 2.4 **Solving Equations** Solutions of equations, algorithms, equations of the form  $y = ax + b$
- 2.5 **A New Pattern** (exponents and compounding) Whole number exponents, exponentiation as an operation, applications of exponents to real-world problems
- 2.6 **Scientific Notation** Scientific notation for abbreviating large and small numbers, positive and negative integral exponents, applications in science
- 2.7 **Making Exponents Work for You** Laws of exponents, integral exponents, applications to algebraic expressions and scientific notation

### Looking Back, Review Exercises

#### Related Topics

**Algebraic Manipulation** Combining like terms, solutions to equations with variables on both sides, fractional terms, exponents, radicals, absolute value, least common multiple

**Inequalities** Solutions to one-variable inequalities and their graphs on number lines, solutions to one-variable absolute value inequalities and their graphs on number lines

**“Star” Operations** Operations, star operations

## Chapter 3: The Algebra of Straight Lines

- 3.1 **Coordinate Systems**  
Rectangular coordinate axes, numerical location of points
- 3.2 **Plotting Sets of Points** Graphical interpretation of inequalities, coordinate descriptions of horizontal and

vertical lines, and segments of regions bounded by them, set-builder notation

- 3.3 **Straightness** Slope, lines described by  $y = mx$ , points on or off lines through the origin
- 3.4 **Exploring Lines with a Graphing Calculator**  
The graphing calculator as a tool for exploring patterns of straight lines, the graphical effect of changing the slope value or adding a constant
- 3.5 **Lines That Do Not Go Through (0, 0)** The slope-intercept form of a linear equation, the algebraic relationship of parallel lines
- 3.6 **Two Points Determine a Line** The two-point derivation of a linear equation, applications to change-of-scale situations such as temperature measurement

### Looking Back, Review Exercises

#### Related Topics

**Coordinate Geometry** The point-slope and standard form of linear equations, changing between slope-intercept, point-slope, and standard form,  $x$ - and  $y$ -intercepts, midpoints of line segments on number lines and coordinate planes, slopes of perpendicular lines

#### Graphing Inequalities in the Coordinate Plane

Solutions to one- and two-variable inequalities and their graphs on coordinate planes

#### Graphing Absolute Value Equations in the Coordinate Plane

Graphs of two-variable linear absolute value equations, graphical effect of changes to the equation  $y = |x|$

## Chapter 4: Estimating with Lines

- 4.1 **Graphs Versus Tables** Scattergrams, comparison of tables and graphs of two-variable data
- 4.2 **Estimating Values Between Data Points: Linear Interpolation** Linear interpolation graphically and algebraically, limitations of the process
- 4.3 **Lines Representing Data Sets** Lines of “best fit” for data sets, the least-squares line
- 4.4 **Least-Squares Lines on a Graphing Calculator** Linear regression with a graphing calculator, correlation coefficient, estimation of unknown data points, applications in science
- 4.5 **Forecasting** Interpolation versus extrapolation, forecasting with regression lines, some limitations of extrapolation

### Looking Back, Review Exercises

#### Related Topics

**Median-Median Line—Another Line of Best Fit** Calculating the median-median line with and without a graphing calculator

**Exploring the Correlation Coefficient of a Set of Linear Data** Relationship between the correlation coefficient and the fit of the regression line to a data set, outliers effect on correlation coefficients

### Book 1b

#### Chapter 5: Using Lines and Equations

- 5.1 **A Guessing Game** *Binary choice strategies, tree diagrams, comparison of algorithms*
- 5.2 **Organizing Trial and Error** *Applications of the guessing-game algorithm, estimation of roots*
- 5.3 **A Picture Is Worth** *Graphical solutions to pairs of linear equations*
- 5.4 **Pictures on Your Calculator** *Isolation of variables in equations, calculator-assisted solutions to pairs of linear equations*
- 5.5 **Precise Answers to Real-Life Situations** *Solutions to equations using intercepts and substitution, interpretation of graphical and algebraic features of equations in applied settings*
- 5.6 **Using Algebra to Solve Systems of Equations** *Algebraic solutions to pairs of linear equations, recognition of coordinates of solution points*
- 5.7 **Pictures for Data** *Data sets that do not fit a linear model, nonlinear graphs for comparing data sets*

#### Looking Back, Review Exercises

##### Related Topics

**Solving Systems of Equations** *Substitution, graphic, and linear combination methods of solving systems of equations*

#### Chapter 6: How Functions Function

- 6.1 **It All Depends** *Functions in real-world situations, the language of functions, functions described by patterns and tables*
- 6.2 **Functions Step by Step** *Functions on finite sets, recursive sequences, the terms of a sequence, equal functions*
- 6.3 **Between the Dots** *Step functions in the real world, restrictions on domains, graphs of functions*
- 6.4 **Describing Functions with Algebra** *Linear equations as functions, images of real numbers, the graphing calculator as a tool for finding images and graphing functions*
- 6.5 **Growth Function** *Exponential functions as models for growth, effects of different growth rates, compounding, evaluation and graphs of exponential functions with a calculator*
- 6.6 **Links in a Chain: Composition of Functions** *Composition of functions, real-world applications of function composition, algebraic calculation of the composite of two functions*

#### Looking Back, Review Exercises

##### Related Topics

**Algebraic Proportions** *Cross multiplication, proportions, percentage problems, unit conversions*

**Sequences** *Arithmetic and geometric sequences recursively and explicitly, explicit form using a graphing calculator*

#### Chapter 7: Counting and Probability

- 7.1 **Counting Sets of Things** *Patterns in counting problems, computer-assisted counting, counting as a function, finite and infinite sets*
- 7.2 **Venn Diagrams: Counting with Pictures** *Venn diagrams, intersection, union, difference, complement, disjoint sets*

#### 7.3 Tree Diagrams and the Fundamental Counting

**Principle** *Uses of tree diagrams, the Fundamental Counting Principle, factorial notation*

#### 7.4 Probability: What Are the Chances

*Probability as the likelihood of an event, probabilities from intuition or experience, experimental probability*

#### 7.5 Equally Likely Outcomes

*Probability as the ratio of acceptable outcomes to total outcomes, the complement of an event, union and intersection of events*

#### 7.6 Using Random Numbers for Simulation

*Random-integer lists on a calculator, design and application of random-integer simulation schemes for various situations*

#### Looking Back, Review Exercises

##### Related Topics

**Geometric Probability** *Probability as ratio of acceptable area to total area in geometric shapes*

**Absolute Value Revisited** *Absolute value inequalities, set-builder notation, interval notation, graphing solutions on number lines and on a graphing calculator*

#### Chapter 8: Quadratic Functions and Equations

#### 8.1 Combining Linear Equations

*Addition and subtraction of linear functions, multiplication and composition of linear functions, quadratic functions, the standard form of a quadratic function*

#### 8.2 The Quadratic Shape

*Gravity described quadratically, first and second differences of a function, parabolas*

#### 8.3 Using Quadratic Equations

*The form of a quadratic equation, quadratic equations in real-world situations, graphical solutions, maximum or minimum values of quadratic functions*

#### 8.4 Solving Quadratic Equations

*Algebraic solutions of various forms of quadratic equations, relationship of algebraic solutions to graphs of quadratics*

#### 8.5 The Quadratic Formula

*The process of completing the square, derivation of the Quadratic Formula, the Quadratic Formula as a method of finding roots, quadratic equations that have no real solutions*

#### Looking Back, Review Exercises

##### Related Topics

**Families of Functions** *Polynomial function graphs, degree of polynomials, even and odd functions, end behavior, turning points*

**Special Factors** *Common monomial factoring, difference of squares, perfect squares, factoring quadratics of the form  $ax^2 + bx + c$*

**Quadratic Inequalities** *Quadratic inequalities on a graphing calculator, solutions to systems of quadratic inequalities on a graphing calculator*

**Year 2:** Starting with basic ways of measuring length and area, **Book 2a** uses symmetries of planar shapes to ask and answer questions about polygonal figures. Algebraic ideas from **Year 1** are elaborated by providing them with geometric interpretations. Scaling opens the door to similarity and then to angular measure, which builds on the concept of slope from **Year 1**. Extensive work with angles and triangles then lays the groundwork for right-angle trigonometry. Principles of congruence and triangulation are employed in innovative ways to show their applicability to the real world. **Book 2b** begins by exploring the properties of circles, and then generalizes the two-dimensional ideas of **Book 2a** to three dimensions, starting with fold-up patterns and contour lines. Coordinate geometry connects this spatial world of three dimensions to the powerful tools of algebra. That connection is then used to explore systems of equations in three variables, extending the treatment of two-variable equations in **Year 1**. In addition, matrices are used to help describe the coordinate geometry of basic transformations.

## Book 2a

### Chapter 1: The Building Blocks of Geometry: Making and Measuring Polygons

- 1.1 **Measuring Lengths** *Mathematical models, globes vs. flat maps of the world, discrete and continuous quantities, units of measure, length measurement*
- 1.2 **Paths, Polygons, and Perimeter** *Polygonal paths, polygons and their standard names, algebraic formulas for perimeters of some polygons*
- 1.3 **Symmetry** *Reflection symmetry, perpendicular bisectors, shortest path to a line*
- 1.4 **Regular Polygons** *Congruent angles, classification of quadrilaterals, regular polygons via symmetry, converse of statements*
- 1.5 **Areas of Right-Angled Figures** *Squares as units of area, finding areas of regions by tiling with square units, formulas for the area of rectangular and right-triangular regions*
- 1.6 **Area and Algebra** *Algebraic identities, diagrams of algebraic area problems, using areas of rectangles to illustrate algebraic laws*
- 1.7 **Triangles and Triangulation** *Constructing triangles from side lengths, constructing altitudes of triangles, formula for triangular area, polygonal areas by triangulation*
- 1.8 **The Pythagorean Theorem** *The Pythagorean Theorem and its converse, finding lengths and areas with the Pythagorean Theorem, constructing right angles with the converse of the Pythagorean Theorem, the distance between two points*
- 1.9 **It Varies with the Square** *Recognizing direct and inverse variation and square variation, algebraic expressions for direct and inverse variation, direct and inverse variation in real-world settings*
- 1.10 **Volume** *Cubes as units of volume, finding volumes of rectangular boxes by filling with unit cubes, formula for the volume of a rectangular box*

#### Looking Back, Review Exercises

#### Related Topics

**Special Points within Triangles** *Orthocenter, incenter, circumcenter, centroid, Euler line*

**Coordinate Geometry**

**Tessellations** *Tessellations, regular tessellations, semi-regular tessellations, isometry, reflection, rotation, reflection, glide reflection*

### Chapter 2: Similarity and Scaling: Growing and Shrinking Carefully

- 2.1 **The Same Shape** *Recognizing similar figures, scaling in maps, models, and blueprints, using ratios and proportions to verify similarity*
- 2.2 **Similar Triangles and Rectangles** *Scaling triangles and rectangles, verifying similar triangles or rectangles using side-length ratios, estimating and computing scaling factors for particular size constraints*
- 2.3 **How to Measure Angles** *Congruent angles, measuring angles using degrees and using slope, describing and measuring angles by rotation*
- 2.4 **Finding Angle Size Efficiently** *Using TAN and TAN<sup>-1</sup> to move between angle measure and degree measure, supplementary and vertical angles*
- 2.5 **Parallel Lines and the Angle Sum of a Triangle** *Corresponding and alternate interior angles, complementary angles, the angle sum of a triangle*
- 2.6 **Parallelograms and Congruent Triangles** *Determining a triangle from two sides and the included angle (SAS), determining the angles of a triangle from its sides (SSS), formula for the area of a parallelogram*
- 2.7 **Other Tests for Congruent Triangles** *Determining a triangle from two angles and one side (ASA, AAS), SSA and AAA as insufficient congruence tests, properties of isosceles triangles*
- 2.8 **Other Polygons** *Triangulation of polygons, sums of interior and exterior angles of polygons, interior and exterior angles of regular polygons*
- 2.9 **Stretching and Shrinking Angles and Areas** *Invariance of angle measure under scaling, recognizing similar triangles by AAA, areas of scaled figures, finding scaling factors*
- 2.10 **Stretching and Shrinking Volumes** *Calculating volumes of scaled figures, finding scaling factors for three-dimensional objects*

#### Looking Back, Review Exercises

#### Related Topics

**Coordinate Geometry and Quadrilaterals**

*Properties of quadrilaterals*

**Constructions** *Construct angle bisectors, circles inscribed in triangles, congruent angles, and quadrilaterals by hand*

**Constructions and Investigations Using Technology** – *Constructions using graphing software*

# Year 2: Books 2a and 2b

## Geometric Concepts in Two and Three Dimensions

### Chapter 3: Introduction to Trigonometry: Tangles with Angles

- 3.1 The Sine of an Acute Angle** *The sine of an acute angle as a ratio of lengths, finding sines with a graphing calculator, applications of the sine function*
- 3.2 The Cosine of an Acute Angle** *The cosine of an acute angle as a ratio of lengths, finding cosines with a graphing calculator, a trigonometric form of the Pythagorean Theorem, the secant and the cosecant of an angle*
- 3.3 The Tangent of an Acute Angle** *The tangent of an acute angle as a ratio of lengths, finding tangents with a graphing calculator, the cotangent of an angle*
- 3.4 Going Backwards: The Inverse Trig Functions** *The inverse functions  $\sin^{-1}x$ ,  $\cos^{-1}x$ , and  $\tan^{-1}x$ , domains of inverse trig functions, calculator-assisted applications of these inverse functions*
- 3.5 Vectors in a Plane** *Describing vectors by magnitude and direction, adding and subtracting of vectors, multiplying a vector by a scalar, applications to directed forces*
- 3.6 Measuring Triangles Using Sines and Cosines** *The Law of Sines and the Law of Cosines and their relation to triangle congruence tests, application to measurement problems*
- 3.7 Sine and Cosine Curves: Going Around in Circles** *Cyclic function patterns in the real world, sine and cosine of angles as points on the unit circle, extend the domains of the trig functions, shape of trig function graphs, circle equations in algebraic and trigonometric form*

#### Looking Back, Review Exercises

#### Related Topics

**Special Right Triangles** *Pythagorean triples, 45-45-90 and 30-60-90 triangles, irrational numbers, reduced radical form*

## Book 2b

### Chapter 4: Circles and Disks

- 4.1 Points of View** *Circles by center-radius definition and symmetry, mathematical terms for features of circles, practical, symbolic, and aesthetic properties of circles*
- 4.2 Points on Circles** *Construct circles through one or two given points and their common features, construct circles through three noncollinear points and find their center and radius, converse of conditional statements, truth of a conditional statement versus truth of its converse*
- 4.3 Drawing Circles With a Graphing Calculator** *Graph circles given a center and radius with a graphing calculator, represent circles by parametric equations, graph circles parametrically, change the size and/or position of circles by changing its equation*
- 4.4 Area and Circumference** *Connection between area and circumference of the unit circle, area and circumference of circles as functions of the radius, find the radius, diameter, circumference, and area of circles from any one of these measures, meaning and origin of  $\pi$*
- 4.5 Pieces of Circles** *Use central angles to find length of circular arcs and area of any sector, area of an annulus and sectors of them*
- 4.6 Inscribed Angles** *Inscribed angles, use measure of central angles to find measure of inscribed angles that intercept the same arc*

- 4.7 Curves of Constant Width** *Construction, perimeter, and area of Reuleaux triangles and other noncircular curves of constant width*

#### Looking Back, Review Exercises

### Chapter 5: Shapes in Space

- 5.1 Another Dimension** *Polyhedra, height of pyramids from edge measurements, cones, height and edge radius of cones from the flat cutout angle*
- 5.2 Three-Space in Layers** *Contour lines on topographical maps, contour lines and cross sections to describe three-dimensional objects in layers, shapes of cylinders and prisms using cross sections, surface area of prisms*
- 5.3 Finding Volumes by Stacking** *Volume of prisms and cylinders, Cavalieri's Principle for volumes*
- 5.4 The Volumes of Spheres, Cones, and Pyramids** *Volume of spheres, cones, and pyramids, connection between volume of spheres, cylinders, and cones*
- 5.5 Turning Around an Axis** *Solid of revolution from planar regions, volumes of various solids of revolution, center of gravity of planar regions bounded by a regular polygon, rectangle, triangle, circle, or semicircle*
- 5.6 Space by the Numbers** *Locations in 3-space using coordinates, distance between two points in 3-space*
- 5.7 Stacks of Lines** *Coordinate descriptions of lines and planes parallel to coordinate planes and axes*
- 5.8 The Algebra of Three-Dimensional Shapes** *Coordinates to describe solid and hollow three-dimensional shapes, inequalities to describe regions in 2-space and 3-space, equations describing circles in 2-space and spheres in 3-space*
- 5.9 Time and Other Dimensions** *Time, temperature, and other measurement scales as coordinate axes, two- and three-dimensional pictures that represent non-spatial measurements, equations and inequalities describing shapes in two- and three-dimensional time-space, our world as a four-dimensional space*

#### Looking Back, Review Exercises

### Chapter 6: Planes, Linear Equations, and Matrices

- 6.1 Graphing Systems of Equations** *Solutions to systems of two equations in two unknowns with a graphing calculator and algebraic methods, geometric solution of systems of two equations in two unknowns, geometric descriptions of equations with three variables, equations for plane surfaces*
- 6.2 Consistent and Inconsistent Systems** *Consistent and inconsistent systems of equations geometrically, systems of two equations as consistent or inconsistent, solutions to systems of two equations by substitution*
- 6.3 Solving Systems of Equations** *Solutions to systems of two and three linear equations by elimination*
- 6.4 Using Matrices to Solve Linear Systems** *Systems of equations as matrices, elementary row operation, reduced row echelon form, Gaussian elimination*
- 6.5 Matrix Operations** *Add and subtract matrices, multiply a matrix by a number, multiply two matrices, identity matrix, matrices representing real-world situations*
- 6.6 Transformations** *Three basic types of isometries, dilations of the plane, matrices to apply linear transformations*

#### Looking Back, Review Exercises

# MATHConnections® Algebra 2

**Year 3:** The unifying theme of this year is the use of mathematical models to describe and solve real-world problems. **Book 3a** investigates polynomial functions, exponential and logarithmic functions, and periodic functions in successive chapters, developing and extending many ideas from Years 1 and 2. The emphasis throughout this material is the utility of mathematical tools for describing and clarifying what we observe in the world around us. This theme continues as both algebraic and geometric tools are applied to optimization techniques, including linear programming. **Book 3b** uses algebraic tools to extend and formalize ideas of combinatorics, probability, and statistics from Year 1 in a variety of modeling settings. Then the idea of modeling itself is examined in some depth by considering the purpose of axioms and axiomatic systems, logic, and mathematical proof. Various forms of logical arguments, already used informally throughout Years 1 and 2, are explained and used to explore algebraic and combinatorial systems, such as modular arithmetic. These logical tools then provide guidance for a mathematical exploration of irrational numbers and infinity, areas in which commonsense intuition is often unreliable. Finally, the axiomatic structure is related to Euclid's plane geometry, connecting his formal system with geometric concepts from Year 2.

## Book 3a

### Chapter 1: Linear, Quadratic, and Other Algebraic Functions

- 1.1 **Making Models** *Mathematical models, linear equations, linear models, perpendicular lines and their slopes, equation for a line given a point and the slope*
  - 1.2 **Quadratic Models** *Simplifying assumptions in mathematical models, domain and range of functions in modeling situations, linear and quadratic models of data sets*
  - 1.3 **The Shape of a Quadratic Function** *Maximum or minimum values of quadratic functions from its graph, parabolas in terms of distance from focus and directrix, reflection properties of parabolas and parabolic surfaces, connect geometric description of parabolas with quadratic functions*
  - 1.4 **Solving Quadratic Equations** *Solutions to quadratic equations graphically and algebraically, discriminant, vertex and the axis of symmetry of quadratic functions, maximization and minimization problems*
  - 1.5 **The Complex Numbers** *Complex numbers, square roots of negative numbers, add, subtract, multiply, and divide complex numbers, complex numbers as points on the complex plane*
  - 1.6 **Transformations of Quadratic Functions** *Transformations of parabolas algebraically, vertex, axis of symmetry, and direction of opening for transformed quadratic functions, quadratic functions to construct models of data, point-slope formula for linear equations*
  - 1.7 **Polynomial Functions** *Maximum or minimum values of cubic functions graphically, cubic functions modeling maximization problems, graphs of cubic functions modeling smooth transition paths, characteristics of polynomial function graphs, turning points and x-intercepts for polynomial functions*
  - 1.8 **Solving Polynomial Equations** *Standard form of polynomial equations, roots of polynomial equations algebraically, add and subtract polynomials, multiply and divide polynomials by binomials, construct a polynomial curve through given points on horizontal lines*
  - 1.9 **Going Backwards: Inverse Functions** *Inverse of functions algebraically and graphically*
  - 1.10 **Reciprocal and Absolute Value Functions** *Relationship between functions and their reciprocals, graphs of reciprocal functions, asymptotes to graph reciprocal functions, absolute value function graphs*
- Looking Back, Review Exercises**

### Chapter 2: Exponential Functions and Logarithms

- 2.1 **Mathematical Explosions** *Exponential functions, features of an exponential curve*
  - 2.2 **Working With Exponents** *Positive, negative, zero, and fractional exponents, translate fractional exponents into other equivalent representations*
  - 2.3 **Exponential Growth** *Compound interest rates, connection between compound interest and exponential growth, features of various exponential graphs, convert compound interest formulas to formulas based on doubling*
  - 2.4 **Exponential Decay** *Exponential growth and exponential decay equations and curves, quantities based on growth and decay functions, relate depreciation equations to equations based on halving*
  - 2.5 **Continuous Growth Models** *Continuous and discrete growth models, limiting values, role of  $e$  in models of growth and decay*
  - 2.6 **Logarithms: Exponents in Reverse** *Inverse of exponential functions as logarithms, logarithms of any positive number with a graphing calculator, estimate logarithms for various bases, logarithms as exponents, log of a product or quotient as a sum or difference of logs, log of a power as a product of a number and a log*
  - 2.7 **Using Logarithms to Analyze Data** *Analyze data with logarithms, data represented by power functions or exponential functions, construct exponential and power functions to fit given data*
  - 2.8 **More Applications of Logarithms** *Solutions to exponential equations with logarithms, logarithms to measure earthquakes, sound intensity, and acidity, Rule of 72*
- Looking Back, Review Exercise**

### Chapter 3: The Trigonometric Functions

- 3.1 **Cyclic Events in the Natural World** *Periodic function graphs, period, periodic functions in the natural world*
- 3.2 **Figuring All the Angles** *Positive, negative, and coterminal angles of rotation, rotation angles in degrees and radians, common rotation angles in degrees and radians, convert radians to degrees and vice versa*
- 3.3 **Generating Circular Functions** *Sine, cosine, and tangent function graphs, maximum, minimum, and period for each function, graphs of each function from circular motions, graph of each function with a graphing calculator, ordered pairs for*

# Year 3: Books 3a and 3b

## Advanced Algebra, Functions, and Modeling.

*each function with a graphing calculator, their graphs, or the unit circle*

**3.4 Transforming the Trigonometric Functions** *Transformed trigonometric functions and their period and amplitude, equations for trigonometric functions from their graphs*

**3.5 Fitting Periodic Functions to Data!** *Fitting appropriate trigonometric function to cyclical data, model cyclic real-world phenomena with periodic functions*

**3.6 Inverses of Trigonometric Functions** *Inverses of the sine, cosine, and tangent functions, graphs of the inverses of each function, graphical relationship between each function and its inverse, inverse functions with a calculator*

**Looking Back, Review Exercises**

### Chapter 4: Optimization: Math Does it Better

**4.1 What is Linear Programming?** *Scheduling problems and their acceptable solutions, blending problems and their acceptable solutions*

**4.2 Linear Programming: A Graphical Approach** *Decision variables, feasible solutions, optimal feasible solutions, and objective functions, linear programming models, feasible region graphs and profit lines to solve linear programming problems in two variables*

**4.3 Linear Programming: An Algebraic Approach** *Slack in linear programming, solutions to linear programming problems with “dictionaries”, solutions to linear programming problems with more than two variables*

**4.4 Transportation Problems** *Feasible solutions to transportation problems, solutions to transportation problems with linear programming, solutions to transportation problems with graphs and cost lines*

**4.5 A Glimpse of Graph Theory: Spanning Trees** *Vertices and edges of graphs, weighted graphs, connected graphs, cycles, and minimal spanning trees to describe real-world problems, solutions to real-world problems with spanning trees*

**4.6 Inequalities and Geometric Programming** *Arithmetic-geometric mean inequality geometrically and algebraically, solutions to optimization problems with arithmetic-geometric mean inequalities*

**Looking Back, Review Exercises**

## Book 3b

### Chapter 5: Counting, Probability, and Statistics

**5.1 Counting in Block Diagrams: What’s the Problem?** *Paths in small block diagrams, sums required to solve small block-diagram problems, quickest routes in small block diagrams, estimate of paths in large block diagrams*

**5.2 Permutations: When Order Counts** *Solutions to counting problems with the Fundamental Counting Principle and permutations, paths in block diagrams, limitations of computers in solutions to large problems*

**5.3 Dynamic Programming** *Solutions to block diagram problems with dynamic programming, solution times required in exhaustive searches versus dynamic programming, working backwards as an efficient problem-solving tool*

**5.4 Combinations: When Order Doesn’t Count** *Solutions to counting problems and block diagram problems with combinations, combination of  $n$  objects taken  $r$  at a time, factorials to count combinations*

**5.5 Probability: How Likely Is It?** *Solutions to problems involving uncertainty with probability, solutions to problems with partial information using conditional probability, independent events, probability of intersection of two events*

**5.6 Binomial Distributions: One or the Other** *Binomial probability distribution, binomial probability algebraically, graphical representation of binomial distributions, relate probability to areas of rectangles*

**5.7 Expected Value** *Expected value, calculate expected values in experiments with a binomial distribution, solutions to problems using expected values*

**5.8 Normal Curves** *Normal curve, relate the normal curve to binomial probabilities, probability with the normal curve, relate probabilities to area under curves, approximate binomial probability with normal probability*

**5.9 Statistical Models** *Analyze sample data to draw conclusions, 95% confidence interval, interpret poll results with confidence intervals, unbiased polls*

**Looking Back, Review Exercises**

### Chapter 6: Logic and Axiomatic Systems: Playing by the Rules

**6.1 Every Game Has Its Own Rules: Axiomatic Systems** *Axioms and theorems and their differences, axiomatic systems, instances of axiomatic systems*

**6.2 Say What You Mean: Writing Good Definitions** *Characteristic definitions, circular and noncircular definitions, writing good definitions*

**6.3 Some Rules About Rules: Basic Logic** *Laws that govern the truth values of standard logic, negations of statements, including negations of quantified statements, effect of and or on truth values, De Morgan’s Laws, hypothesis, conclusion, and converse of conditional statements, biconditional statements*

**6.4 Coming to Conclusions: Direct and Indirect Proofs** *Direct and indirect proofs, counter examples, justify statements with direct and indirect proofs, contrapositive and inverse of conditional statements*

**6.5 How Are Axioms Chosen?: Mathematical Modeling** *Mathematical models, inductive and deductive reasoning, advantages and disadvantages of mathematical models, Law of the Lever, state axioms for a group*

**6.6 Instances of a Group** *Explain axioms for a group, use group axioms to describe properties of integers and rational numbers, modular addition for positive integers*

**6.7 The Power of Proof** *Identity elements, inverses, and cancellation laws in operation tables, explain proofs of some theorems about groups, symmetries of polygons as permutations, combine symmetries by function composition, groups of symmetries*

**6.8 Number Systems, Large and Small** *Structure of the real number system using the field axioms, inverses to define subtraction and division by addition and multiplication, relationship of group axioms and field axioms, construct finite number systems with modular arithmetic*

**Looking Back, Review Exercises**

*continued on next page*

## Chapter 7: Infinity—The Final Frontier?

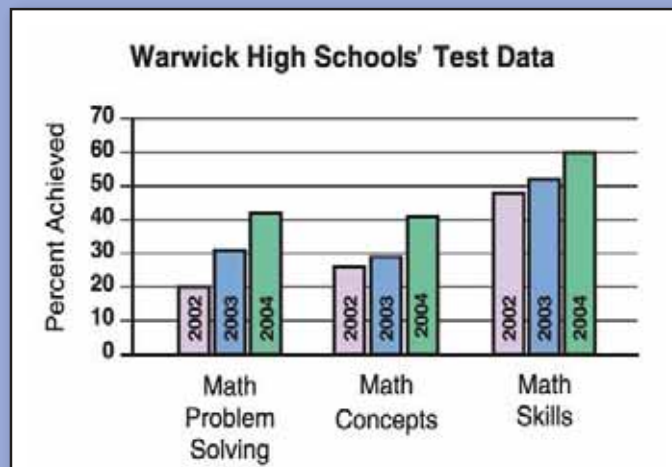
- 7.1 A Brief History of Infinity** *Role played by infinity in previously studied mathematics, understanding the concept of infinite processes*
- 7.2 The Natural Numbers and Proof by Induction** *Natural numbers as an infinite set, proof of some conjectures with induction*
- 7.3 Rational Number Properties** *Historical development of the set of natural numbers, integers, and rational numbers, classify numbers in the number sets, relationship among the number sets, group properties of the number sets*
- 7.4 The Rational Number System: Bounded Infinity** *Rational numbers on the number line in fractional and decimal form, rational fractions to decimal form, repeating decimals as the result of infinite processes*
- 7.5 Irrational Numbers and the Real Number System** *Irrational numbers, irrational numbers on the number line in decimal and radical form, relationship among the sets of rational and irrational numbers to the set of real numbers*
- 7.6 Infinite Series** *Evaluate infinite series with exhaustion, understanding of limits, repeating decimals to rational fractions, find the sum of infinite geometric series*
- 7.7 Finding Area Under a Curve** *Area under curves by approximation, understanding of integration, area under curves as the limit of an infinite process, area under a curve with a graphing calculator*
- 7.8 Countable Sets and Cardinality** *The different sizes of infinity, equivalent sets by a one-to-one correspondence between them, differences between the infinite characteristics of the sets of rational numbers, irrational numbers, and the real numbers*

### Looking Back, Review Exercises

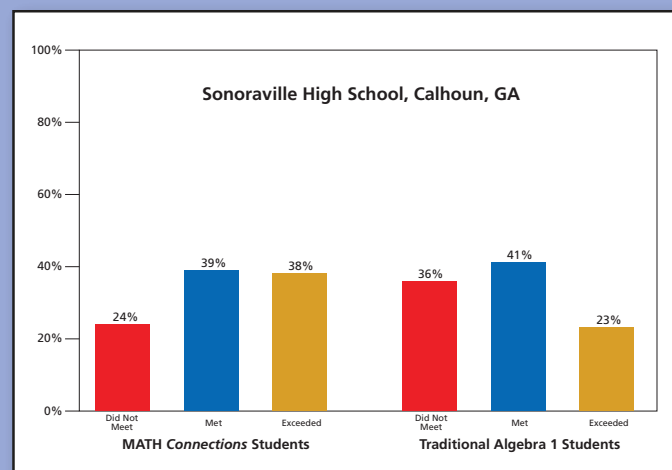
## Chapter 8: Axioms, Geometry, and Choice

- 8.1 Consistency: Can Any Statement Be an Axiom?** *Consistent axiomatic systems, axiomatic systems describing real-world situations*
- 8.2 The Geometry of Euclid** *Historical importance of Euclid's work, The Elements, supply reasons for steps in some of Euclid's proofs, draw diagrams that accompany Euclid's arguments, relate statements in Euclid's Book I to geometric ideas*
- 8.3 Euclid and Parallel Lines** *Relationship of Euclid's Fifth Postulate and parallel lines, prove some of Euclid's propositions, logical connections between parallel lines, the angle sum of triangles, and squares*
- 8.4 Independence: Can There Be Too Many Axioms?** *Dependent and independent axioms, prove axioms of systems are independent*
- 8.5 The World Through Different Eyes: Non-Euclidean Geometries** *Parallel Postulate resulting in different systems of geometry, true geometric facts in all of these geometries, basic facts of Euclidean geometry that are false in the non-Euclidean geometries, instances of non-Euclidean geometry*
- 8.6 Mathematics and Our World** *The connection of the different geometries to our world*

### Looking Back, Review Exercises



On a statewide math test in Warwick, Rhode Island, over 1000 students from all three Warwick high schools showed significant increases in achievement since the implementation of MATH Connections. Math skills increased 12%; math concepts increased 15%; and math problem-solving increased 21%.



In Calhoun, Georgia MATH Connections students scored higher than students in a traditional algebra class.

# Supplemental Material

## Outcome-based

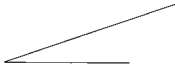
## Quiz 2.3

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

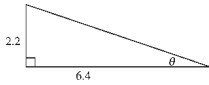
1. Circle the angles that are congruent. (2 pts)



2. Give the measure of the angle below, and then use your protractor to construct a congruent angle. (2 pts)



Use the drawing below for problems 3 and 4.



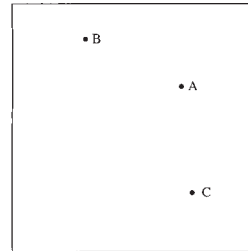
3. Determine the measure of angle  $\theta$  in degrees. (2 pts)
4. Determine the measure of angle  $\theta$  in slope measure. (2 pts)
5. A standard clock shows the time is 2:00. Determine the measure of the acute angle that separates the minute and the hour hand. (1 pt)

## Chapter 3 Test

## Outcome-based Variation B

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

1. In the diagram below, use a ruler to construct a set of coordinate axes with the origin at point B and use a scale of  $1/2''$  for each axis. (2 pts)



2. Using the coordinate system in question 1, give the coordinates of: (2 pts)

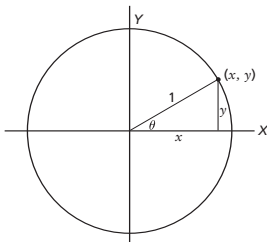
point A: \_\_\_\_\_ point C: \_\_\_\_\_

3. On the coordinate system in question 1, plot and label the following points: (2 pts)

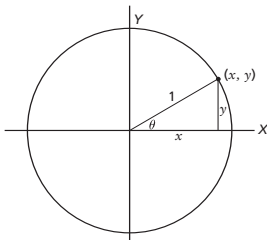
D: (1, -2) E: (-2, 1)

## Graphs and Tables

## Chapter 4



The unit circle  
Display 4.17



The unit circle  
Display 4.17  
(p. 312)

## Variation A

## Quiz 8.3

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

1. You have a six-sided cube numbered 2, 4, 6, 8, 10, 12. Write the sample space for rolling the cube once.
2. You roll two cubes that are numbered the same as in problem 1.  
a) A table describing the sample space for the sums when rolling these two cubes is started below. Please complete the table.

+	2	4	6	8	10	12
2					12	
4				12		
6						
8	10					
10						
12		16				

- b) What is the probability of obtaining a sum of 14? \_\_\_\_\_
- c) What is the probability of obtaining a sum of 24? \_\_\_\_\_
- d) What is the probability of obtaining a sum of 11? \_\_\_\_\_
- e) What is the probability of obtaining an even sum? \_\_\_\_\_
- f) What is the probability of obtaining a sum of 16 and at least one cube shows a 6? \_\_\_\_\_
3. If you were designing a fair game using eight-sided solids, what would be your directions?







