NAME: DATE:

The Language of Probability

Probability

PROBABILITY is a measure of the chance or likelihood that an event will occur.

What are the chances:

That rain will fall in Oakland tomorrow?

That you will win the lottery?

That you will roll a sum of 6 with two dice?

THE PURPOSE OF THIS SECTION IS TO:

- Understand and use the language of probability
- Use numbers to express probability and odds

The Language of Probability

The **probability** or **chance** of an **event** happening can be expressed as a number ranging from 0 to 1. Probability can also be expressed as a percentage ranging from 0% to 100%.

If an event is **impossible,** its probability is defined to be 0, or 0%.
If an event is **certain,** its probability is defined to be 1, or 100%.

If an event has an equal chance of happening as not happening, we use the term **equally likely**. The probability is defined to be $\frac{1}{2}$ or 50%.

The probability of an event can be shown and described along a continuum or number line:

The phrase **less likely** means there is a lesser chance of the event happening than not happening. The probability of that event falls between 0 and $\frac{1}{2}$ on the number line.

The phrase **more likely** means that there is a greater chance of the event happening than not happening. The probability of that event falls between $\frac{1}{2}$ and 1 on the number line.

Practice Using the Language of Probability

1. Use the language of **impossible**, **less likely**, **equally likely**, **more likely**, and **certain** to describe the probability of these events. For each description, explain your thinking.

EXAMPLE

a. Winning the lottery if 1,000,000 people play.

This event is less likely to happen because the chance of winning is very small.

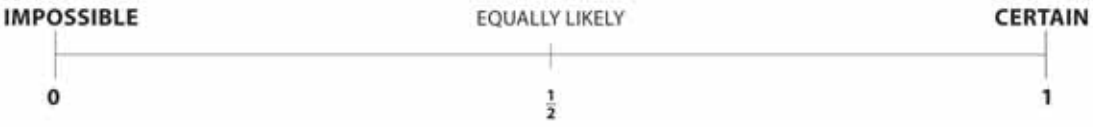
b. Choosing a green marble from a bag that contains one green marble and one red marble.

c. The sun won't rise in the East.

d. Choosing a green marble from a bag that contains only red marbles.

e. A penny thrown in a lake will sink.

2. Use the letters **a** through **e** to place the probabilities of the events above on the number line.



Explain how you decided where to place event **a**.

3. Use the language of **impossible**, **less likely**, **equally likely**, **more likely**, and **certain** to describe the probability of these events. For each description, explain your thinking.

   a. Choosing a green marble from a bag that has 8 green marbles and 20 red marbles.

   b. The trees will walk to school tomorrow.

   c. The sun will set tonight.

   d. Choosing a red card from a deck that has 26 red cards and 26 black cards.

   e. One or more students will be absent from class tomorrow.

4. Use the letters **a** through **e** to place the probabilities of the events above on the number line.

Explain how you decided where to place event **e**.

5. Use the language of **impossible**, **less likely**, **equally likely**, **more likely**, and **certain** to describe the probability of these events. For each, explain your thinking.

   a. Choosing a green marble from a bag that has 3 green marbles and 4 red marbles.

   b. Snow falling in the low desert in August.

   c. Tossing a coin and having it land tails-side up.

   d. Tossing a coin three times and having it land tails-side up every time.

   e. Getting a yellow gumball out of a gumball machine containing only yellow gumballs.

6. Use the letters **a** through **e** to place the probabilities of the events above on the number line.

7. Make up three events of your own and place them on the number line to indicate their probabilities.

   a. _____

   b. _____

   c. _____

Calculating Probability

The probability of an event can be described using the words **impossible**, **less likely**, **equally likely**, **more likely**, and **certain.** For some events, it is possible to calculate the exact value of the probability and express it numerically.

For example: A bag contains 5 green marbles and 10 white marbles.
One marble will be chosen from the bag at random, so each of the marbles has an equal chance of being chosen.

Since there are 15 marbles in the bag, there are _____ **possible outcomes.**
The set of all possible outcomes is called the **sample space**:

Green marbles 
and
White marbles 

To find the probability of drawing a green marble, first identify the **favorable outcomes** in the sample space. In this case, each of the green marbles represents a favorable outcome.
The probability of choosing a green marble is equal to the ratio of the number of favorable outcomes to the total number of possible outcomes.

$$P(\text{green}) = \frac{\text{Number of green marbles}}{\text{Total number of marbles}} = \frac{5}{15}$$

The notation *P*(green) stands for "the probability of choosing a green marble."

In general, the formula for the probability of an **event** is:

$$\textbf{P(event)} = \frac{\text{Number of favorable outcomes in the sample space}}{\text{Number of possible outcomes in the sample space}}$$

1. Complete the steps below to find the probability of drawing a white marble.

   a. Circle the **favorable** outcomes in the sample space.

   Green marbles 
   and
   White marbles 

   b. How many **favorable** outcomes are there? _____

   c. How many **possible** outcomes are there? _____

   d. $P(\text{white}) =$ _____

Tossing Coins

Tossing a coin results in either heads or tails.

$$P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}}$$

1. Label the sample space.

2. Find the probability the coin will land tails-side up.

   a. How many possible outcomes are there? _____

   b. How many favorable outcomes are there? _____

   c. $P(\text{tails}) =$ ____

3. a. Circle the term that describes the probability of the coin landing tails-side up.

   impossible **less likely** **equally likely** **more likely** **certain**

   b. Explain why you chose that description.

Rolling Dice

Luiza rolls one die. Find the probability of each of the events below.

$$P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}}$$

1. Find the probability of Luiza rolling a 3.

   a. Show the sample space.

   b. Circle the favorable outcomes.

   c. $P(3) = \frac{\square}{\square}$

This is read:
"The probability of rolling a 3 is $\frac{1}{6}$." Or
"The probability of rolling a 3 is one out of six."

2. Find the probability of Luiza rolling a 5. _____

3. Find the probability of Luiza rolling a 7. _____

4. Find the probability of Luiza rolling an even number.

   a. Show the sample space.

   b. Circle the favorable outcomes.

   c. $P(\text{even}) = \frac{\square}{\square}$

5. Find the probability of Luiza rolling an odd number. Write the fraction in simplest form. $\frac{\square}{\square}$

Probability Practice

Probabilities can also be written as decimals or percents.

1. Find the numerical probability of rolling a prime number using one die.

   a. Draw or list the sample space.

   b. Circle the favorable outcomes.

   c. P(prime) = _____

   d. Write this probability as a fraction in simplest form. _____

   e. Express this probability as a decimal. _____

   f. Express this probability as a percent. _____

   Multiply the decimal by 100%.

2. Find the numerical probability of the spinner landing on 3 on the spinner shown to the right.

   a. List the sample space.

   { ___ , ___ , ___ , ___ }

   Brackets are used to indicate the sample space.

   b. How many favorable outcomes are there? _____

   c. P(3) = _____

   d. Express this probability as a decimal. _____

   e. Express this probability as a percent. _____

   f. What is the numerical probability of the spinner landing on 1? _____
   Explain how you know.

Spinners

1. Label the spinner so that the chance of the spinner landing on a 4 is certain.

   a. List the sample space:

   { ____ , ____ , ____ , ____ }

   b. $P(4)$ = _____

   c. Express this probability as a decimal. _____

   d. Express this probability as a percent. _____

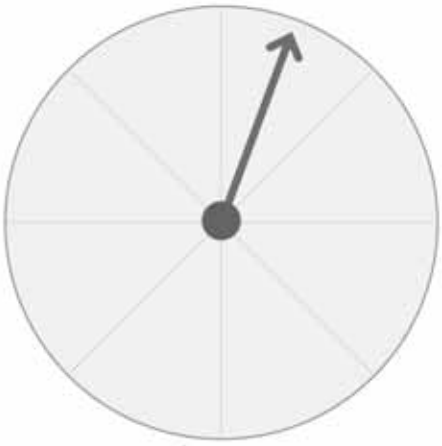
   e. $P(3)$ = _____

2. This is the sample space of the spinner to the right.

   {A, A, B, C, D, E, E, E}

   a. Label the spinner appropriately.

   b. $P(E)$ = _____
   Explain how you know.



Each portion of the spinner is the same size.

Marbles

1. A bag contains some red marbles and some green marbles.
   There are a total of 10 marbles in the bag.
   The probability of choosing a red marble from the bag is $\frac{3}{10}$.

   a. Draw the sample space.

   Explain how you know this is the sample space.

   b. How would the sample space change if we added 2 more green marbles to the bag before choosing one?

   c. How many possible outcomes are now in the sample space? _____

   d. If we want to find $P(\text{red})$, how many favorable outcomes are now in the sample space? _____

   e. What is the probability of choosing a red marble from this bag that now has 12 marbles? _____

2. Another bag has a total of 20 marbles. Some marbles are red and some marbles are green. The probability of choosing a red marble from this bag is also $\frac{3}{10}$. Use this probability to answer the following questions.

   a. How many red marbles are in the bag? _____

   b. How many green marbles are in the bag? _____

   c. What is the probability of choosing a green marble? _____

   d. What is the probability of **not** choosing a green marble? _____

   e. What is the sum $P(\text{red}) + P(\text{green})$? _____

   f. What is the sum $P(\text{red}) + P(\text{not red})$? _____

When the Sum Equals 1

For the bag of red marbles and green marbles discussed above, the probabilities are:

PROBABILITY OF FAVORABLE OUTCOME (RED)	PROBABILITY OF UNFAVORABLE OUTCOME (GREEN)	PROBABILITY OF ALL OUTCOMES (RED OR GREEN)
$P(\text{red}) = \frac{3}{10}$	$P(\text{green}) = \frac{7}{10}$	$P(\text{red}) + P(\text{green}) = \frac{10}{10} = 1$

In any situation, the sum of the probabilities of all the possible outcomes is 1.

$$P(\text{red}) + P(\text{green}) = 1$$

The sum of the probability of an event happening and the probability of the event not happening is also 1.

$$P(\text{green}) + P(\text{not green}) = 1$$

$$P(\text{red}) + P(\text{not red}) = 1$$

More Marbles

1. A bag contains red marbles and green marbles.
The probability of choosing a green marble out of the bag is $\frac{4}{15}$.

This means there are 4 green marbles for every 15 marbles in the bag.

$$\frac{4}{15} + \frac{?}{15} = 1$$

   a. What is the probability of choosing a red marble? _____

   b. What information does this probability tell us about the number of marbles in this bag?

2. Another bag of red marbles and green marbles contains 30 total marbles. The probability of choosing a green marble out of this bag is $\frac{4}{15}$.

   a. Show the sample space.

   b. How many green marbles are in this bag? _____

   c. How many red marbles are in this bag? _____

   d. Use your response to part c to find $P(\text{red})$.

   e. Think of another way to determine the probability of choosing a red marble from the bag.

Changing the Number of Marbles

A bag contains red marbles and green marbles.

There are 45 marbles in the bag, and $P(\text{green}) = \frac{4}{15}$.

The probability of $\frac{4}{15}$ can be used to determine how many marbles of each color are in the bag.

To find the number of green marbles, find a fraction equivalent to $\frac{4}{15}$ that has a denominator of 45.

Since $15 \cdot 3 = 45$, multiply $\frac{4}{15}$ by $\frac{3}{3}$.

When you multiply a fraction by one $\left(\frac{3}{3}\right)$, the ratio of green marbles to the total number of marbles stays the same.

The resulting fraction $\frac{12}{45}$ indicates there are 12 green marbles in the bag of 45 total marbles.

1. How many red marbles are in this bag? _____

2. $P(\text{red}) =$ _____

Check your answer: Does $P(\text{green}) + P(\text{red}) = 1$?

3. Another bag contains a total of 18 marbles.
   This bag has 3 colors of marbles: red, green, and yellow.
   The probabilities of selecting a particular marble from this bag are:

$$P(\text{red}) = \frac{1}{6} \qquad P(\text{green}) = \frac{1}{3}$$

   a. How many green marbles are in this bag? _____

   b. How many red marbles are in this bag? _____

   c. How many yellow marbles are in this bag? _____

   d. $P(\text{yellow}) =$ _____

   e. $P(\text{not yellow}) =$ _____

   f. Check your answer to part d by finding the sum of the three probabilities $P(\text{green})$, $P(\text{red})$, and $P(\text{yellow})$.

Practice with Marbles

1. Jasmine and Faith have a bag filled with green marbles, blue marbles, and red marbles. There are a total of 10 marbles in the bag.
   $P(\text{green}) = \frac{1}{10}$ and $P(\text{blue}) = \frac{3}{10}$.

   a. Draw and label each of the marbles in this bag.

   b. How many red marbles are in this bag? _____

   c. What is the probability of drawing a red marble from this bag? _____

   d. Write $P(\text{red})$ as a percent. _____

   e. Write $P(\text{blue})$ as a percent. _____

   f. Write $P(\text{green})$ as a percent. _____

2. Jasmine and Faith were asked to create another bag with the same probabilities. This new bag must contain a total of 30 marbles.

   a. How many red marbles should they put in this bag? _____

   b. How many blue marbles should they put in this bag? _____

   c. How many green marbles should they put in this bag? _____

   d. Complete this chart to show that the probabilities are the same as the probabilities for the bag with 10 marbles.

	As a fraction	As a percent
$P(\text{red})$		
$P(\text{blue})$		
$P(\text{green})$		

The Odds Are...

Faith wants the carnival committee to have turtle races at the carnival again.
In this game, five players each choose a turtle to run the race.
Each turtle has a letter (A through E) attached to its back.
Each turtle has an equal chance of winning.

Faith wants to know if you can determine the chances of any one turtle winning.
Carlos says that in horse races on TV, the chances are called "the odds."
Instead of comparing the number of favorable outcomes to the number of *possible* outcomes,
the odds compare the number of favorable outcomes to the number of *unfavorable* outcomes.

1. Faith wants to find the **odds** that Turtle B will win the race.
   a. List the sample space. _____
   b. How many possible outcomes are there? _____
   c. How many favorable outcomes are there? _____
   d. How many unfavorable outcomes are there? _____
   e. The odds in favor would be _____ to _____.
      Turtle B (favorable) Turtles A, C, D, E (unfavorable)
   f. Explain how the odds of Turtle B winning are different from the probability of Turtle B winning.

2. In a different version of turtle races, the **probability** of Turtle B winning is $\frac{1}{8}$.
   Each turtle still has an equal chance of winning.
   a. List the sample space. _____
   b. How many turtles are in this race? _____
   c. Draw a circle around the favorable outcomes in the sample space above.
   d. Draw a square around the unfavorable outcomes in the sample space above.
   e. What are the odds of Turtle B winning this race? _____

3. The odds of Turtle B winning another race are 1 to 11.
   What is the **probability** that Turtle B would win this race? _____
   Explain how you know.