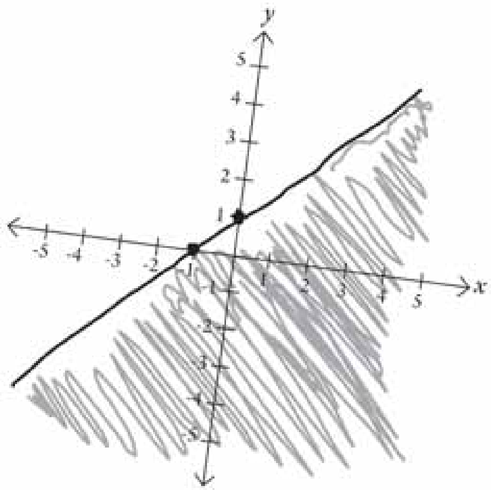
NAME: DATE:

Inequalities in Two Variables

The Language of Probability



THE PURPOSE OF THIS SECTION IS TO:

- Use graphic representations to visualize the solutions to inequalities in two variables
- Understand that solutions to inequalities in two variables are usually regions in the plane with or without the boundary lines

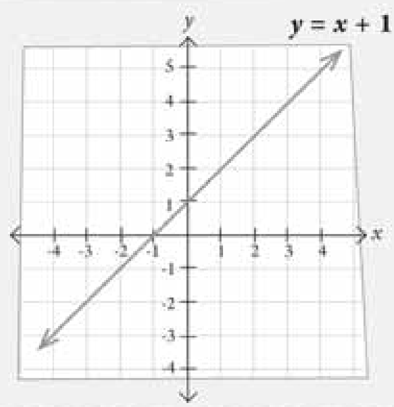
MATERIALS YOU WILL NEED:

- Straightedge

Solutions to Inequalities in Two Variables

The solution to an equation in two variables is the set of all ordered pairs (x, y) that make the equation true. For example, the line graphed on the grid to the right represents the solution set for the equation $y = x + 1$.

Similarly, the solution to an inequality in two variables is the set of all ordered pairs (x, y) that make the inequality true.



1. Plot these ordered pairs on the graph above:

   (1, 4) (2, 3) (4, -3) (-3, -2) (-1, -3) (-2, 0) (1, 0) (0, 1)

2. Circle the ordered pairs above that are solutions to the equation $y = x + 1$.

3. Which of the ordered pairs above are solutions to the inequality $y < x + 1$?

   (___ , ___) (___ , ___) (___ , ___)

   Circle these points on the graph.

4. What do you notice about where these points lie on the graph?

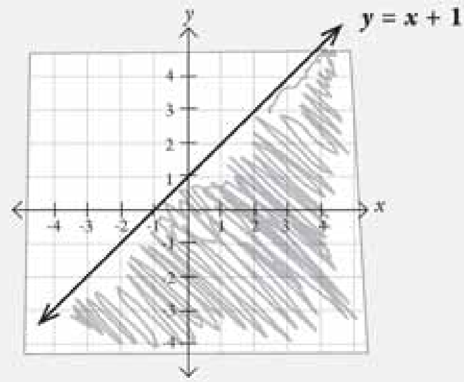
If $x = 1$ and $y = 4$, is $y < x + 1$?

Graphs of Solutions to Inequalities in Two Variables

The graph of the solution set for an inequality in two variables is a region of the coordinate plane

The shaded region and the solid line shown on the graph to the right represent the solution set for the inequality $y \le x + 1$. All the ordered pairs (x, y) in this region and on the solid line make the inequality $y \le x + 1$ a true statement.

The line $y = x + 1$ is called the **boundary line** for the inequality.



Use the graph above to answer the following questions.

1. Write an ordered pair that lies in the region of the solution set. ____________________
   Plot this point on the graph.

2. Write an ordered pair that does not lie in the region of the solution set. ____________
   Plot this point on the graph.

3. Circle the ordered pairs (x, y) that are solutions to the inequality $y < x + 1$.

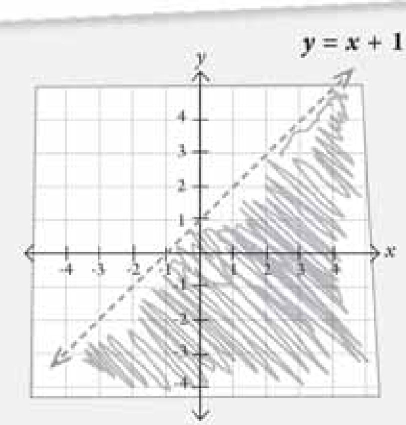
   (1, 4)   (2, 3)   (-3, -2)   (0, 1)   (-1, -3)   (-2, 0)   (1, 0)

4. How do you think the solution set for the inequality $y < x + 1$ differs from the solution set for the inequality $y \le x + 1$?

The solution set for the inequality $y < x + 1$ is shown on the graph to the right.

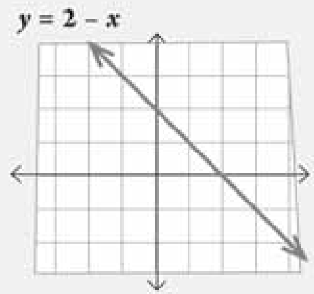
The solution set for this inequality does not include the points on the line $y = x + 1$ because the inequality is strictly less than.

On the graph, this is shown by a dashed boundary line.



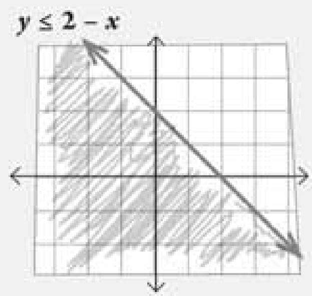
Summary

The boundary line for an inequality divides the coordinate plane into two **regions**.

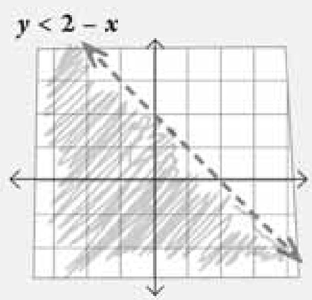


The graph of the **solution set for a linear inequality in two variables** will be one or the other of these regions, and may or may not include the boundary line.

- The region **includes the boundary line** if the inequality symbol is ≤ or ≥ (as indicated by a solid line).



- The region **does not include the boundary line** if the inequality symbol is a strict inequality < or > (as indicated by a dashed line).



Practice Graphing Inequalities

Complete the tables for each of the following inequalities.

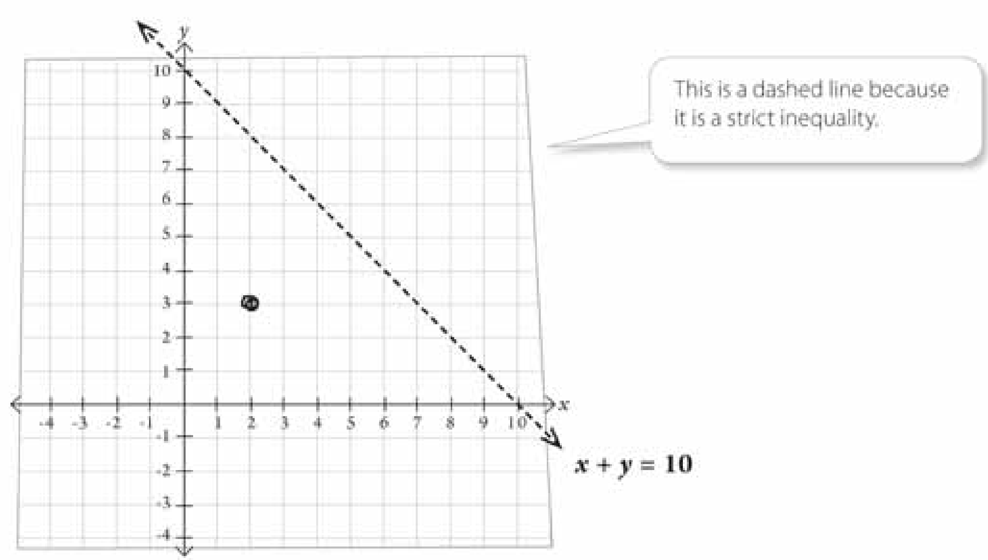
1. $x + y < 10$

a. Which of the following ordered pairs are solutions to the inequality?

Ordered pairs	Is a solution	Is not a solution	Why?
(2, 3)	✓		2+3=5, which is less than 10
(7, 8)		✓	7+8=15, which is not less than 10
(-4, 10)			
(4, 6)			
(1, -4)			
(10, 2)			
(20, -9)			
(0, 0)			

b. Plot the ordered pairs that are solutions to the inequality $x + y < 10$.
Use the coordinate plane below.

c. Shade the region that represents the solution set for the inequality $x + y < 10$.



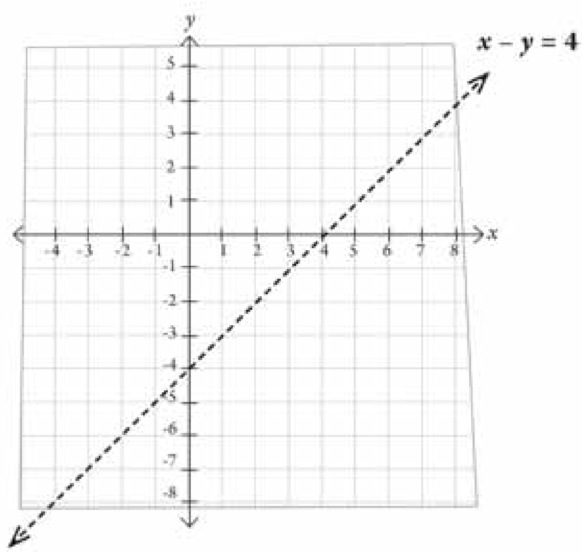
2. $x - y > 4$

a. Which of the following ordered pairs are solutions to the inequality?

Ordered pairs	Is a solution	Is not a solution
(4, 0)		
(6, 1)		
(-2, -7)		
(6, 2)		
(7, -1)		
(0, 0)		

b. Plot the ordered pairs that are solutions to the inequality $x - y > 4$.
Use the coordinate plane below.

c. Shade the region that represents the solution set for the inequality $x - y > 4$.



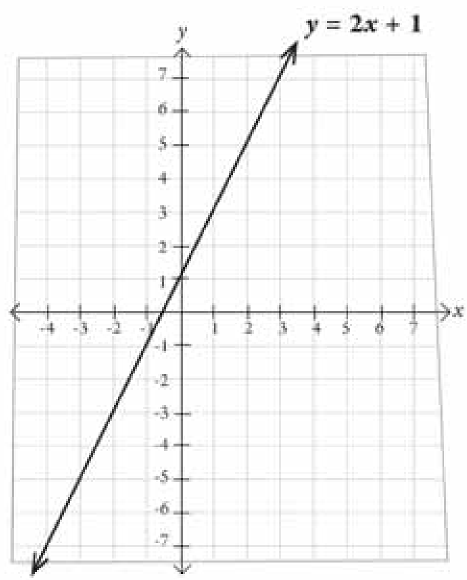
3. $y \geq 2x + 1$

a. Which of the following ordered pairs are solutions to the inequality?

Ordered pairs	Is a solution	Is not a solution
(1, 0)		
(0, 1)		
(2, 0)		
(-2, 3)		
(3, 7)		
(0, 0)		

b. Plot the ordered pairs that are solutions to the inequality $y \geq 2x + 1$. Use the coordinate plane below.

c. Shade the region that represents the solution set for the inequality $y \geq 2x + 1$.



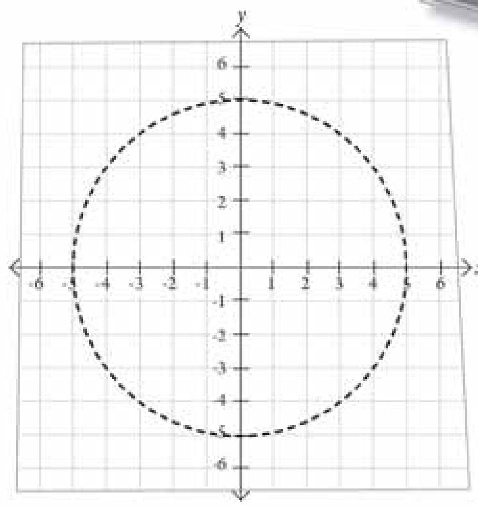
Inside or Outside the Circle?

The graph of the solution set for a linear inequality is the region on one side of a line.

Now consider an inequality where the boundary is not a straight line.

The circle graphed to the right also divides the plane into two regions: the region inside the circle and the region outside the circle.

This circle is the graph of the equation $x^2 + y^2 = 25$:



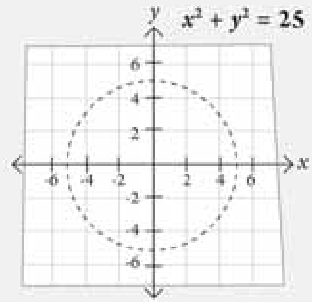
1. Which of the following ordered pairs are solutions to the inequality $x^2 + y^2 > 25$? Plot the points as you answer the questions.

Ordered pairs	$x^2 + y^2$	Is a solution?	Inside, outside, or on the circle?
(1, 2)	$1^2 + 2^2 = 5$	No	Inside
(-3, 6)	$(-3)^2 + 6^2 = 45$	Yes	Outside
(5, 0)			
(1, 5)			
(-3, -4)			
(0, 0)			
(0, -6)			

2. Shade in the region that is the solution set for the inequality $x^2 + y^2 > 25$.

Using a Single Test Point

The circle $x^2 + y^2 = 25$ divides the plane into two regions: **inside** the circle and **outside** the circle.



- We can find which region is the solution to an inequality by using a single test point that does not lie on the circle.
- If the test point makes the inequality statement true, the region would be the one which **includes** this point.
- If the test point makes the inequality statement false, the region would be the one which **does not include** this point.

Use the test point $(0, 0)$ to determine the region of the solution set for the inequality $x^2 + y^2 \leq 25$:

First substitute:

$$x^2 + y^2 \leq 25$$

$$0^2 + 0^2 \leq 25$$

$$0 \leq 25 \quad \text{TRUE}$$

Since you can choose any test point not on the boundary line, choose one with simple calculations.

Since $0^2 + 0^2$ is less than 25, the point $(0, 0)$ is in the solution set.

Since $(0, 0)$ is inside the circle, the solution set must be the entire region inside the circle.

The inequality $\leq$ includes =, so the points on the boundary line are included in the solution set.

Use the test point $(4, 1)$ to find the solution sets for the following inequalities.

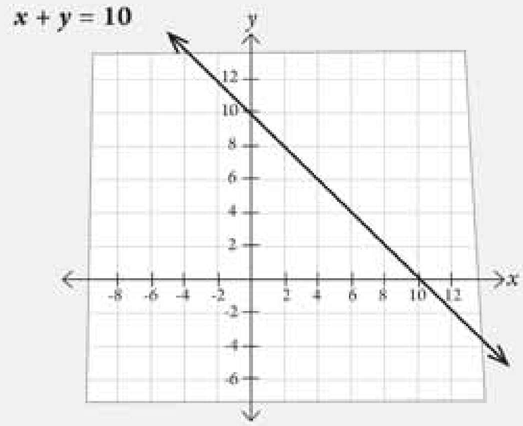
	$x^2 + y^2 \leq 25$	$x^2 + y^2 > 25$
Is the test point $(4, 1)$ a solution?	Yes, because $4^2 + 1^2$ is less than 25.	No, because $4^2 + 1^2 = 17$, which is not > 25.
Is the solution set inside the circle or outside the circle?		
Is the boundary line (the equation $x^2 + y^2 = 25$) included in the solution set?		

A Test Point

A single test point can be used for linear inequalities.

Consider the inequality $x + y < 10$.

The equation $x + y = 10$ is graphed to the right.



Use the test point $(0, 0)$ to determine the region of the solution set for the inequality $x + y < 10$.

First substitute:

$$x + y < 10$$

$$0 + 0 < 10$$

$$0 < 10 \qquad \text{TRUE}$$

Since $0 + 0$ is less than 10, the point $(0, 0)$ is in the solution set.

Since $(0, 0)$ is below the line, the solution set must be the region that includes $(0, 0)$. So the solution set is the region below the line.

The points on the boundary line are not included because this is a strict inequality $(<)$.

Use the test point $(2, 4)$ to find the solution sets for the following inequalities.

	$x + y > 10$	$x + y \leq 10$
Is the test point $(2, 4)$ a solution?		
Is the solution set below the line or above the line?		
Is the boundary line (the equation $x + y = 10$) included in the solution set?		

Graphing Solution Sets

Find the solution to the following inequality: $2x + 5y \geq 100$

First, we need to graph the equation of the boundary line: $2x + 5y = 100$
Use the values of $x = 0$ and $y = 0$ to find the intercepts of the line:

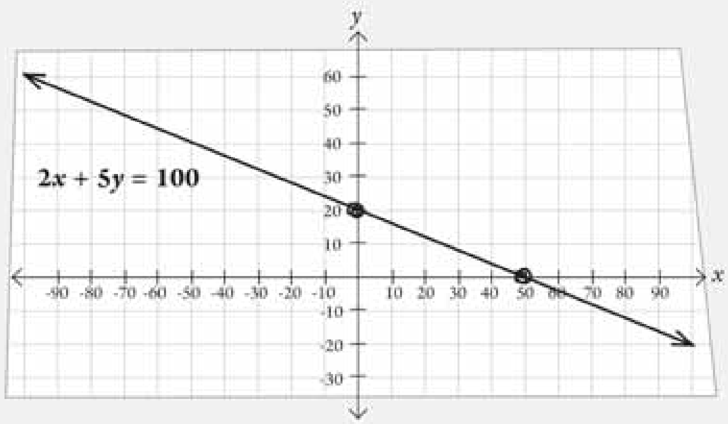
If $x = 0$, then: $2(0)+5y = 100$
$5y = 100$
$y = 20$

This tells us that the point (0, 20) is on the line.

If $y = 0$, then: $2x+5(0) = 100$
$2x = 100$
$x = 50$

This tells us that the point (50, 0) is on the line.

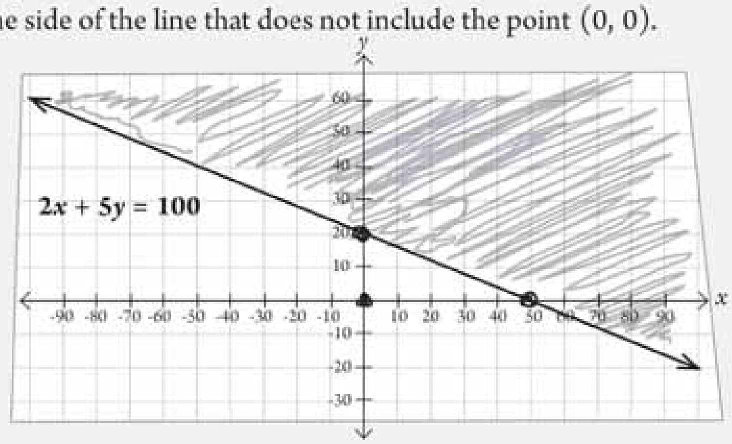
Draw a solid line because the inequality is "greater than or equal to."



Now use the test point (0, 0) and determine if it is in the solution set or not:

$2x + 5y \geq 100$
$2(0) + 5(0) \geq 100$ FALSE

Since zero is not greater than or equal to 100, the point (0, 0) is not in the solution set. The solution set is the side of the line that does not include the point (0, 0).



Practice Graphing Solution Sets

Graph the solution sets for the following inequalities.

Indicate with a **solid** line if the boundary line is included in the solution set.

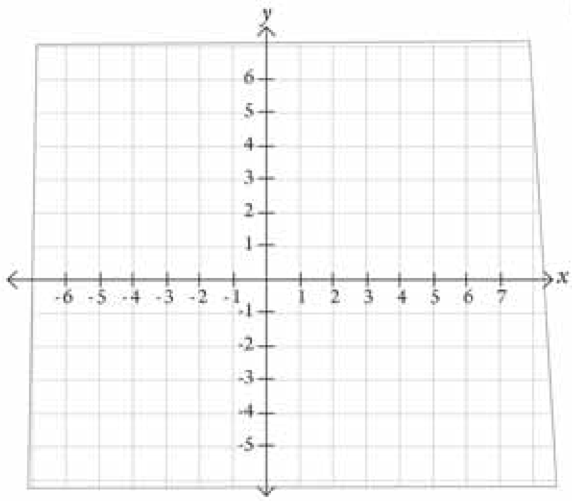
Indicate with a **dashed** line if the boundary line is not included in the solution set.

Shade the region which makes the inequality true, and label the boundary line.

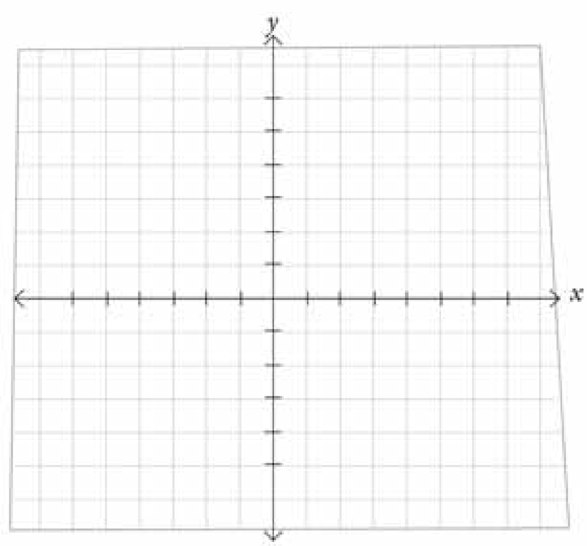
STRATEGY

1. Graph the equation of the boundary line.
2. Determine if the boundary line is included in the solution.
3. Choose a test point and determine if it is in the solution set or not.
4. Shade the solution set region.

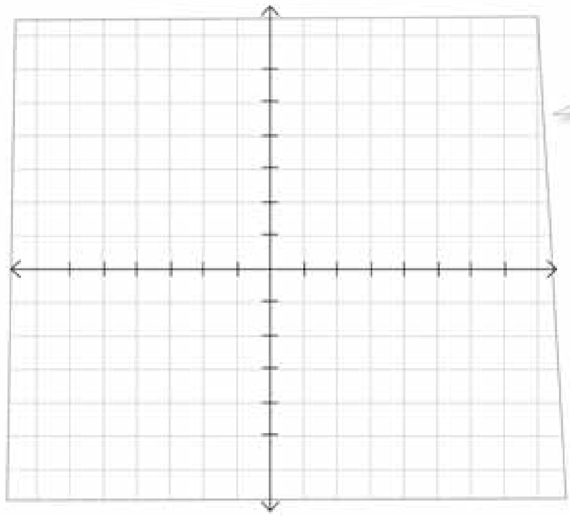
1. $y > 4 - x$



2. $x + y \leq 6$



3. $x + y < 0$



The point (0, 0) cannot be used as a test point here because it lies on the line. It won't tell you which side the solution region is on.

4. $12 > 2x - 3y$