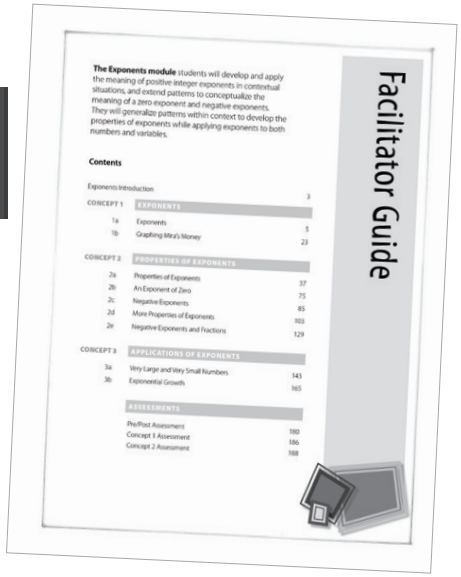
Aim for Algebra

Implementation Guide

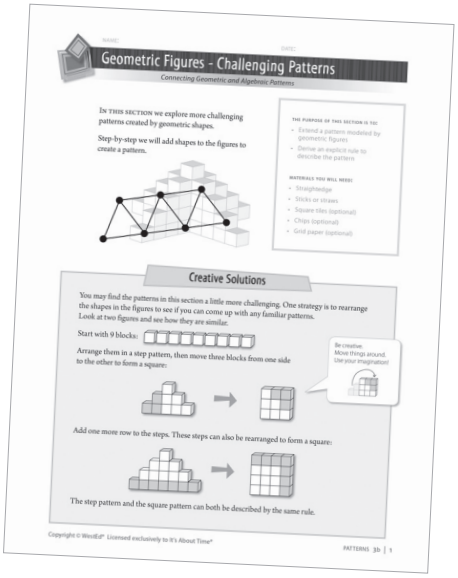


The *Aim for Algebra* curriculum supports instructors as they plan, design, and craft lessons to teach mathematics. A guide is provided for each lesson, in each module of the program.

This Implementation Guide offers an overview of *Aim for Algebra* and instructional strategies to assist the facilitators as they navigate the mathematics in the modules. This guide includes information regarding the structure and intent of lessons, visual maps, and explanations of the component parts of both the student pages and the corresponding Facilitator Guides.

CONTENTS

INTRODUCTION TO THE MATERIALS


Geometric Figures - Challenging Patterns

Connecting Geometric and Algebraic Patterns

In this section we explore more challenging patterns created by geometric shapes.

Step-by-step we will add shapes to the figures to create a pattern.

Creative Solutions

You may find the patterns in this section a little more challenging. One strategy is to rearrange the shapes in the figures to see if you can come up with any familiar patterns. Look at two figures and see how they are similar.

Start with 9 blocks:

Arrange them in a step pattern, then move three blocks from one side to the other to form a square:

Be creative. Move things around. Use your imagination!

Add one more row to the steps. These steps can also be rearranged to form a square:

The step pattern and the square pattern can both be described by the same rule.

The materials you will need:
- Straightedge
- Sticks or straws
- Square tiles (optional)
- Chips (optional)
- Grid paper (optional)

The purpose of this section is to:
- Extend a pattern modeled by geometric figures
- Derive an explicit rule to describe the pattern

The *Aim for Algebra* curriculum materials are intended to assist and support students who face barriers to their success in traditional algebra courses.

Research-based, *Aim for Algebra,* is designed to target specific curriculum areas to help students better understand math content through learning experiences that reinforce, refresh, or reteach important concepts.

Aim for Algebra's instructional program reflects eight essential elements for intervention materials. These elements are:

- Targeted, not comprehensive CURRICULUM
- CONCEPTUALLY based and standards ALIGNED
- FLEXIBLE format and structure
- Embedded INSTRUCTION
- Purposeful task DEVELOPMENT and SCAFFOLDING
- Precise, academic LANGUAGE and concrete MODELS
- ASSESSMENT:
  - pre/post module assessments
  - end of concept assessments
  - formative assessment embedded in the instruction
- Facilitator guides for instructor SUPPORT

Aim for Algebra instructional materials are focused on the development of conceptual understanding rather than rote memorization of skills or procedures. Students are asked to make sense of the mathematics they are experiencing through discussion and written explanations.

To provide students equal access to more difficult concepts, *Aim for Algebra* activities are scaffolded with respect to their placement in the sequence of the modules and within the tasks themselves. This necessitates fidelity to the sequencing and content of the lessons included in the *Aim for Algebra* modules. That is, to ensure the building of understanding, the lessons within a module should be taught in the order presented, without supplemental tasks inserted into the flow of the lessons.

In addition, *Aim for Algebra* is not a "pull out" program, so tasks should not be removed out of order and presented to students. The sequence is critical due to the development of language and mathematical ideas from Concept 1 through Concept 3, and then extending and connecting these foundational prerequisite ideas to algebraic concepts.

The standards-aligned *Aim for Algebra* modules develop understanding of the mathematical concepts and skills that are prerequisites for algebra, over time, through multiple opportunities for students to engage in reasoning, problem solving, inquiry, and discourse.

The Facilitator Guides are designed to provide support as teachers guide students through the *Aim for Algebra* modules while taking advantage of all the built-in tools. Below is a visual map of the structure, order, and scaffolding that is part of each Facilitator Guide page.

The *PURPOSE* statement for each lesson precedes the teaching sections. Each lesson contains three sections: *LAUNCH*, *EXPLORE*, and *SUMMARIZE*. The right column contains additional ideas, suggestions, and mathematical information for facilitating the lessons.

TITLE

Corresponds to the student page title, and reflects the purpose of each page.

SUGGESTED USE CALLOUT

How to use the lesson: guided practice, group work, homework, or assessment.

PAGE THUMBNAIL

Each Facilitator Guide page corresponds directly to the student materials, illustrated here. The number in the diamond corresponds to the page number in the student page.

PURPOSE

Describes the focus of the lesson.

LAUNCH

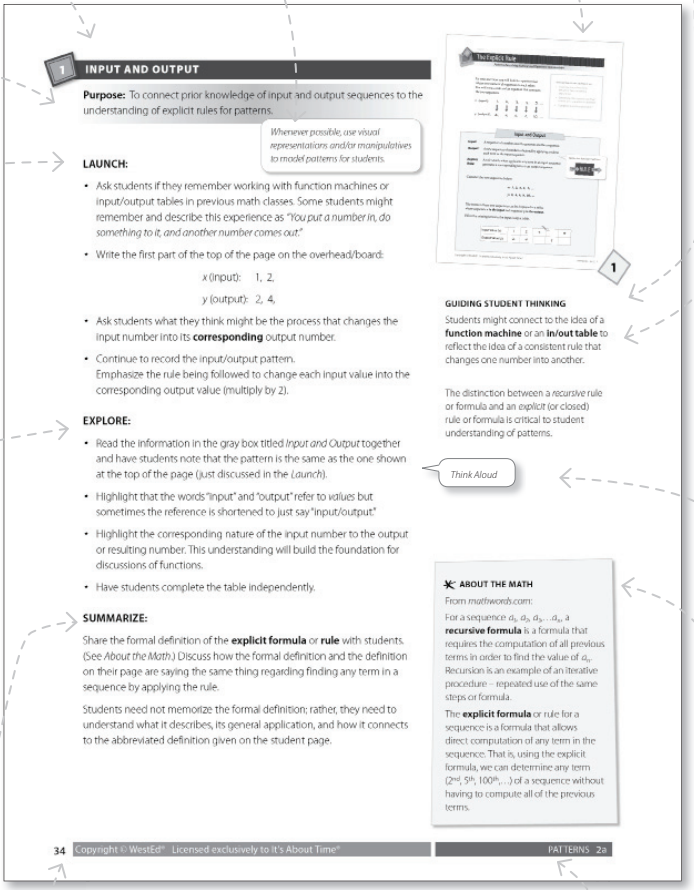
Begins the discussion and connects the content to prior learning; the *Launch* discussion/task might be considered a warm up or anticipatory set, occasionally engaged in prior to giving students their printed materials.

EXPLORE

Choreographs the activities of the lesson, suggests questions to ask students and how to facilitate the groupings and student presentations to engage students in discussions of the math ideas.

SUMMARIZE

Ensures students connect their experiences of the task to the stated *Purpose*. It is critical to save time for students to reflect on and reinforce the ideas presented and promote retention of those ideas. The *Summarize* also might provide extension tasks for discussion and/or homework.



PAGE NUMBER

Indicates the location of each lesson within the Facilitator Guide. This number does not correspond to the associated student page.

FOOTER

Contains the Module name, Concept, Section, and page number.

GUIDING STUDENT THINKING

Includes suggestions for directing student thinking to focus on the important ideas of the lesson; indicates important concepts for students to understand prior to or during the course of each lesson; includes common student misconceptions and ways to redirect students towards correct mathematical thinking.

POSSIBLE STUDENT RESPONSES

Includes ideas students might suggest during the lesson, and questions and answers for responding to student thinking shared throughout the lesson.

THINK ALOUD

Teachers and students say aloud what they are thinking as they perform these tasks.

ABOUT THE MATH

Includes important mathematical concepts for the teacher, including definitions of mathematical terms, explanations of mathematical notation, and information necessary to help students better understand the concepts.

The student pages have several elements to accomodate the necessary focus of instruction. Icons and shaded portions of a page indicate the way students will interact with the mathematics and the tasks. Each student page is designed for guided practice or self-direction to support teachers as they craft their lessons.

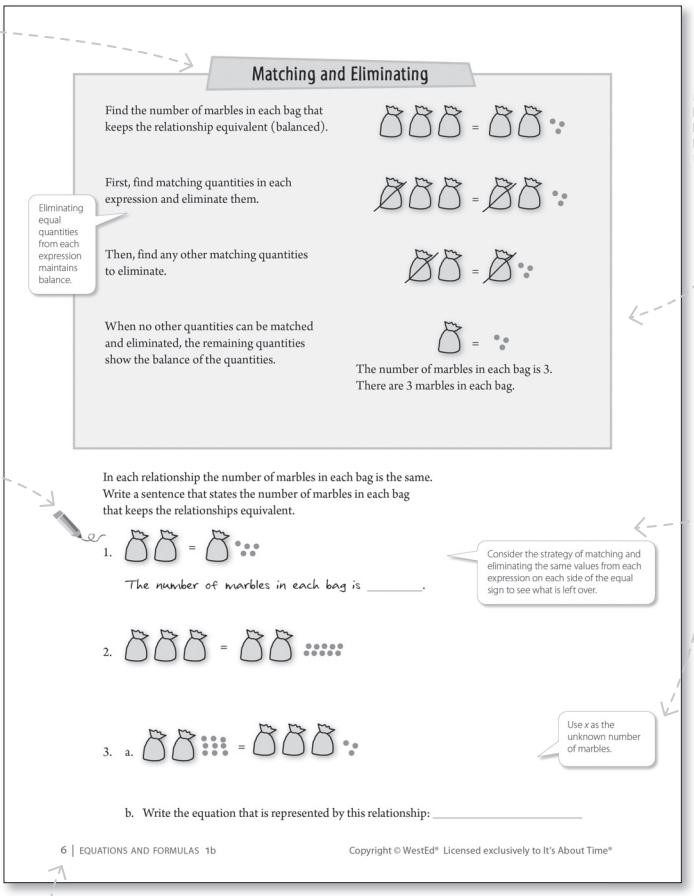
These built-in tools include the intentional scaffolding of the questions, such that there is immediate access for student engagement and the questions become increasingly more complex as the task continues. Below is a visual map of one of the student pages identifying the logic for the structure and organization of the math lesson.

TITLE

Title of student page.

PENCIL

Indicates where students are expected to complete a task.



GRAY BOXES

Indicate instruction with white inserts for student participation.

CALLOUTS

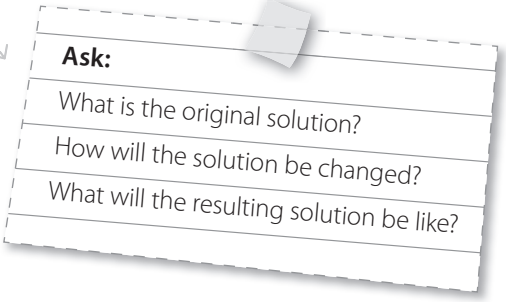
Direct and cue students as they engage in the mathematics.

FOOTER

Contains the Module name, Concept, Section, and page number.

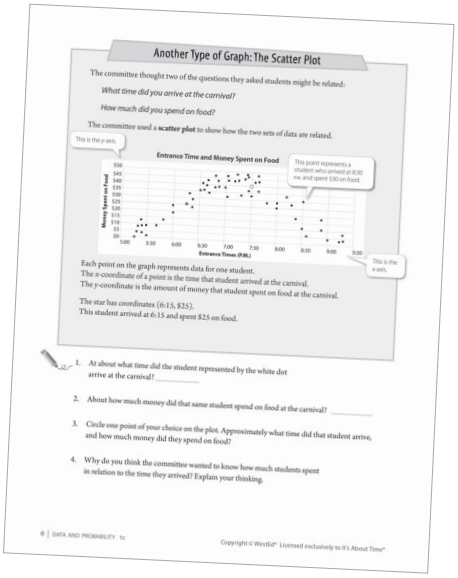
NOTEPADS

Remind students of previous information.



STRUCTURE OF A LESSON

Purpose: To familiarize the facilitator with the general outline of an *Aim for Algebra* lesson. To describe each component of a lesson.


Another Type of Graph: The Scatter Plot

The committee thought two of the questions they asked students might be related:

What time did you arrive at the carnival?

How much did you spend on food?

The committee used a **scatter plot** to show how the two sets of data are related.

This is the y-axis.

Entrance Time and Money Spent on Food

This point represents a student who arrived at 8:30 PM and spent $30 on food.

This is the x-axis.

Each point on the graph represents data for one student.
The *x*-coordinate of a point is the time that student arrived at the carnival.
The *y*-coordinate is the amount of money that student spent on food at the carnival.

The star has coordinates (6:15, $25).
This student arrived at 6:15 and spent $25 on food.

1. At about what time did the student represented by the white dot arrive at the carnival? _____
2. About how much money did that same student spend on food at the carnival? _____
3. Circle one point of your choice on the plot. Approximately what time did that student arrive, and how much money did they spend on food?
4. Why do you think the committee wanted to know how much students spent in relation to the time they arrived? Explain your thinking.

6 | DATA AND PROBABILITY: 7c

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LAUNCH:

- Students may be asked to do an introductory (warm-up or related) task, such as the tasks presented in the *Launch* portion of the lesson guide.
- As the *Launch* concludes, bring the group together for questions, discussion and a check for understanding before proceeding to the *Explore* activities.
    - This group discussion will give students the opportunity to ask new questions and share thinking after the introductory activity.

EXPLORE:

- During the *Explore*, students continue to discuss and complete tasks, independently, in pairs or whole group.
    - Circulate among the groups as students work on the tasks, asking probing questions to ensure they are on the right track.
    - Avoid telling or showing students how to solve the tasks, or how to set up a "procedure" that leads them to solve the task. Instead, emphasize the conceptual nature of the task and the relationships between the concepts. The goal is for *students* to do the thinking and problem solving.
    - It is important to ask probing questions that guide students and do not give away the answer or explicitly suggest a solution to the tasks.

SUMMARIZE:

- Engage students in the *Summarize* segment of the lesson as a reflection, an extension, or another check for understanding. A whole group discussion about specific activities will reinforce the stated *Purpose* of the lesson.
    - Check on students' understanding of the task by asking several students to explain what they know and what they are expected to do for that particular task. Encourage students to *think aloud* as they describe their process. As students describe their process probe them to reveal their reasoning.

Think Aloud

FACILITATING *AIM FOR ALGEBRA*

Purpose: To acquaint facilitators with the development, content, and pedagogy behind the Facilitator Guides for optimum facilitation of *Aim for Algebra*.

Aim for Algebra provides a Facilitator Guide for each module. Each Facilitator Guide provides a lesson plan and mathematical information for each student page. Although the Facilitator Guide is presented in a direct, almost scripted format, facilitation ideas and style are flexible and should reflect your teaching style, while keeping the purpose of the task as the primary focus.

The lesson guides have been developed and tested by teachers in classrooms and have proven successful with students. However, you should feel free to adjust the plan as necessary to meet the needs of your students.

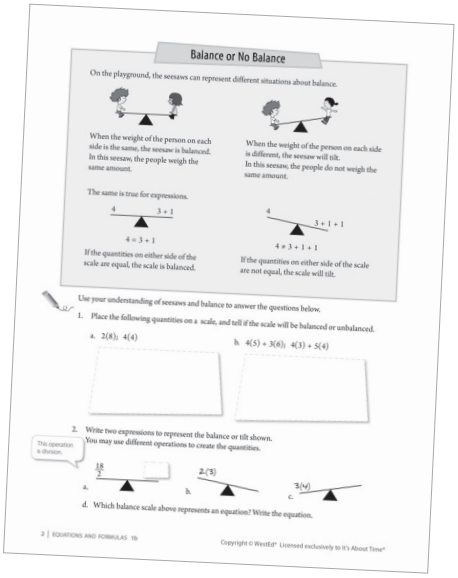
PREPARING TO FACILITATE:

Prior to teaching the material, it is recommended that you read and solve the tasks using as many different approaches as possible, in anticipation of the variety of student responses that might be offered. Consider the tasks both from a facilitator's and a learner's point of view.

This preparation will enable you to be aware of various strategies for approaching tasks, as well as predict possible misconceptions students might have about the content. This awareness will be helpful to understand and follow students' thinking as they explain their work.

Experiencing the tasks as a learner will also allow you to feel confident as you *think aloud* while modeling examples. (See *Guiding Student Thinking*.) As you work through an example, or task, talk aloud about your thinking and the steps you are following, emphasizing the reasoning behind each step you have chosen to follow as you solve the task.

As you work, consider possible connections that might be made to the prior knowledge of your students. Whenever possible provide visual aids and demonstrations to access prior knowledge and clarify connections to the concept. These strategies are particularly helpful for visual learners and English language learners.



Balance or No Balance

On the playground, the seesaws can represent different situations about balance.

When the weight of the person on each side is the same, the seesaw is balanced. In this seesaw, the people weigh the same amount.

When the weight of the person on each side is different, the seesaw will tilt. In this seesaw, the people do not weigh the same amount.

The same is true for expressions.

$4 = 3 + 1$

If the quantities on either side of the scale are equal, the scale is balanced.

$4 \neq 3 + 1 + 1$

If the quantities on either side of the scale are not equal, the scale will tilt.

Use your understanding of seesaws and balance to answer the questions below.

1. Place the following quantities on a scale, and tell if the scale will be balanced or unbalanced.
   a. $2(8)$; $4(4)$
   b. $4(5) + 3(6)$; $4(3) + 5(4)$
2. Write two expressions to represent the balance or tilt shown. You may use different operations to create the quantities.

This operation is division.

$\frac{18}{2}$

$2(3)$

$3(4)$

d. Which balance scale above represents an equation? Write the equation.

2 | EQUATIONS AND FORMULAS 1b

GUIDING STUDENT THINKING

This *think aloud* procedure is the same as suggested for use by teachers as they explain and model the process of solving problems.

Always *ask* students what they are thinking and how they are thinking about a concept *before* explaining, suggesting, or telling them how to proceed.

- It is important to have students explain their thinking before assuming they are making an error or have a misconception.
- When working with a student, look for the essence of the mathematics and conceptual understanding rather than a 'pat' or expected response.
- Encourage variety and inventiveness of approaches to problem solving and a variety of correct representations of solutions.

POSSIBLE STUDENT RESPONSES

If students have difficulty getting started suggest they consider these suggestions from Newman's Error Analysis (Clarke, 1988):

- What is the question asking you to do?
- How are you going to find the answer?
- Do the work to get the answer and tell about your thinking as you work.
- Write down the answer and decide if it makes sense as an answer to the question.

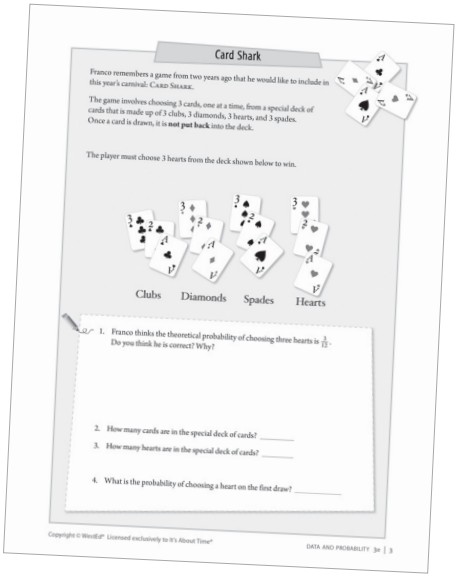
FACILITATING *AIM FOR ALGEBRA*, CONTINUED

Purpose: To acquaint facilitators with the development, content, and pedagogy behind the Facilitator Guides for optimum facilitation of *Aim for Algebra*.

FACILITATING THE TASKS:

- The facilitation/presentation style of the lessons closely resembles the lesson style observed in Japanese classrooms as reflected by the TIMSS and TIMSS-R video studies.
- As a general procedure, tasks are intended to be introduced as a whole group discussion (guided instruction). Students are then directed to spend **individual time** (between 1 and 5 minutes) to make sense of the task.
    - **Individual time** allows students to become familiar with and engage in specific tasks before being asked to share observations, questions, or possible solutions. Students are more likely to participate in the discussion when they feel they know something about the task.
    - Processing time for each student varies, so it is important that students have time and access to each task from the beginning of the discussion, particularly if student materials have not yet been distributed.
    - Tasks might be displayed on an overhead/board/chart paper, allowing students to track the discussion and match it to their own pages. To facilitate this, students might use highlighters to assist them as they track written instructions.
    - Allow students time to read and understand the question for themselves, and then to begin to find a solution in a way that makes sense to them and satisfies the context.
    - After students have had individual time to make sense of the material, they will be ready for individual work time or group work.

It is advisable that the structure of each lesson contain frequent variations. A variety of instructional techniques and strategies can be integrated, every 10 to 20 minutes, to help maintain student engagement and interaction. Suggestions for possible instructional strategies are included in the Facilitator Guides. These structures and strategies include teacher led discussions, guided instruction, or guided practice, and student led discussions. In addition, students should have the opportunity to share with the whole group, and/or work individually, in pairs, or small groups. Students should expect to *think aloud* as they share, put their work on the overhead/board, or create posters so others can view their process.


Card Shark

Franco remembers a game from two years ago that he would like to include in this year's carnival: CARD SHARK.

The game involves choosing 3 cards, one at a time, from a special deck of cards that is made up of 3 clubs, 3 diamonds, 3 hearts, and 3 spades. Once a card is drawn, it is **not put back** into the deck.

The player must choose 3 hearts from the deck shown below to win.

Clubs Diamonds Spades Hearts

1. Franco thinks the theoretical probability of choosing three hearts is $\frac{1}{12}$. Do you think he is correct? Why?

2. How many cards are in the special deck of cards? _____

3. How many hearts are in the special deck of cards? _____

4. What is the probability of choosing a heart on the first draw? _____

GUIDING STUDENT THINKING

A study of NAEP data states, "mathematics students whose teachers emphasize higher-order thinking skills outperform their peers by about 40% of a grade level...students whose teachers conduct hands-on learning activities outperform their peers by more than 70% of a grade level." (Wenglinksy, 2000)

Think Aloud

EXPECTATIONS FOR STUDENTS: WHAT STUDENTS WILL DO

Purpose: To deepen student understanding of the mathematics experienced.

Aim for Algebra attempts to help students develop the "habits of mind" necessary for success in the mathematics classroom.

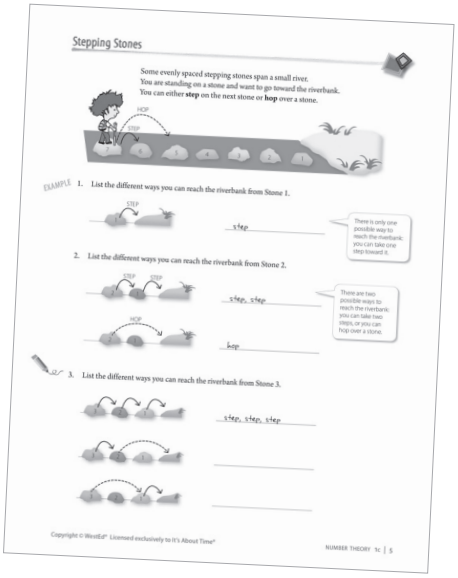
HABITS OF MIND

- High-order thinking skills and questioning strategies: teachers are encouraged to "ask" before you "tell" when instructing students or responding to questions.
- Student participation: all students should be expected to join in the discovery and sharing of information.
- Discourse: students are expected to discuss the mathematics with partners, small groups, or with the whole group and the teacher.
- *Think alouds*: describe the thinking as you or a student models their work.
- Individual time: students need time to make sense of the information for themselves before they will have the confidence to share with others.

To develop these Habits of Mind, the norms for the *Aim for Algebra* mathematics classroom include the following **expectations for students**:

- Share and justify their solution in the context of the tasks
- Explain their thinking and reasoning to others and on paper
- Make sense of other students' explanations
- Ask questions of the teacher or other students for clarification
- Use correct, precise mathematical vocabulary, language, and symbols in their explanations
    - Understanding the precise academic vocabulary will become clear as each lesson develops. Students use the vocabulary in context, so it is not necessary or recommended to teach a separate mini-lesson on the vocabulary itself.

There is a frequent variation of structure for the lessons. The suggestions given in the Facilitator Guides can include teacher led discussions, guided instruction, or guided practice. Students have the opportunity to share with the whole group, and/or work individually, in pairs, or small groups.


Stepping Stones

Some evenly spaced stepping stones span a small river.
You are standing on a stone and want to go toward the riverbank.
You can either **step** on the next stone or **hop** over a stone.

EXAMPLE 1. List the different ways you can reach the riverbank from Stone 1.

step

There is only one possible way to reach the riverbank: you can take one step toward it.

2. List the different ways you can reach the riverbank from Stone 2.

step, step

hop

There are two possible ways to reach the riverbank: you can take two steps, or you can hop over a stone.

3. List the different ways you can reach the riverbank from Stone 3.

step, step, step

GUIDING STUDENT THINKING

Encourage students to present their work on the overhead/board and *think aloud* as they describe their reasoning and justify their solutions.

Remind students to focus on the **process** they use to solve a task, rather than the calculations they perform. Students may not recognize that the answer is only significant for this particular task, but the process is what will transfer to other tasks.

After listening to student thinking, ask questions that will guide students toward discovering and understanding any misconceptions or errors.
Ask open-ended questions to help students clarify their thinking and understandings.

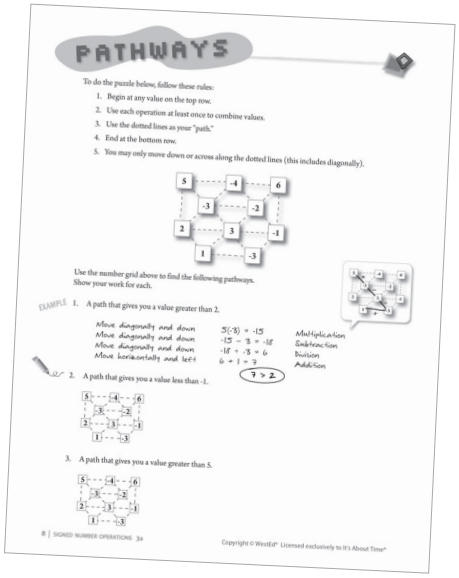
Possible questions to help uncover a student's thinking include:

- What do you know so far?
- What do you need to do?
- Can you tell me how you are thinking about that?
- Can you tell me how you solved the problem, step by step?
- Can you use different numbers to show the steps you discovered?
- How might you explain your process to another student?
- Is there another way you could represent the solution?

These questions can support students to make concrete connections among mathematical ideas.

ABOUT THE MODULES

Purpose: To emphasize the importance of teaching the modules with fidelity.


PATHWAYS

To do the puzzle below, follow these rules:

1. Begin at any value on the top row.
2. Use each operation at least once to combine values.
3. Use the dotted lines as your "path."
4. End at the bottom row.
5. You may only move down or across along the dotted lines (this includes diagonally).

Use the number grid above to find the following pathways. Show your work for each.

EXAMPLE 1. A path that gives you a value greater than 2.

Move diagonally and down — 5(-3) = -15 — Multiplication
Move diagonally and down — -15 − 3 = -18 — Subtraction
Move diagonally and down — -18 ÷ -3 = 6 — Division
Move horizontally and left — 6 + 1 = 7 — Addition
7 > 2

2. A path that gives you a value less than -1.

3. A path that gives you a value greater than 5.

8 | SIGNED NUMBER OPERATIONS 3a

The *Aim for Algebra* curriculum is flexible and can be used as a complete program, or can be implemented as individual modules to support the needs of the students.

The modules are intended to be taught with fidelity. That is, the activities in each module must be taught sequentially in the order presented. *Aim for Algebra* is not a "pull-out-the-tasks-we-like" program, and no additional practice pages or textbook pages need to be used during the course of the module instruction.

The modules are formed by three component parts called concepts. Within each concept, the lessons are organized into sections, representing experiences with a coherent mathematical idea. The sections allow students to engage, experience, and assimilate optimum bite-size amounts of information before going deeper in the mathematical idea.

CONCEPT 1

Connect to prior knowledge.

Begins by developing common language and understanding for the concept through connections to prior learning.

CONCEPT 2

Explores the prerequisite concepts for the algebraic idea through instruction and tasks that approach the math idea in a different way than traditional texts, to fill gaps and reteach misconceptions in a conceptual way and through doing.

CONCEPT 3

Extends the math idea to address algebra standards, to help students make connections to core curriculum and mandated tests.

If the post test results indicate students require additional practice, allow them time for the math ideas to assimilate before introducing supplemental practice and encourage them to use the conceptual approaches presented in the modules when they interact with the mathematical processes.

ASSESSMENTS

Aim for Algebra encourages the use of specific tasks, identified in the Facilitator Guides, as embedded assessment. At the end of Concept 1 and Concept 2, a short assessment is provided as an additional check for understanding of the math content in that concept.

In addition, there is a pre/post test for each module that reflects not only the learning experiences in the module but also released items from local and national mandated tests to ensure students can apply their understanding beyond this program.

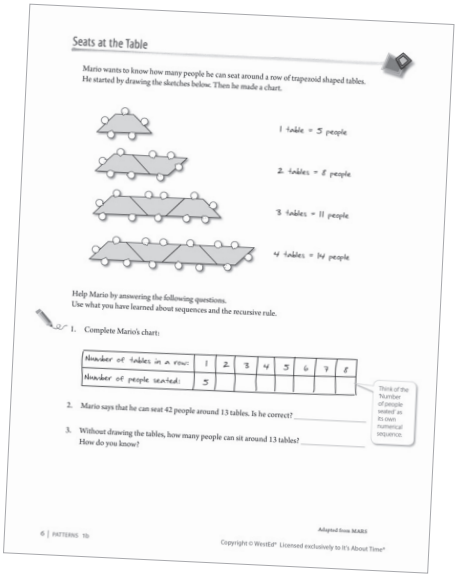
ABOUT THE MATH

A note about calculators:
Calculators are recommended as a tool for problem solving.

Calculator use is intended to allow students to focus on the math concepts and reasoning, rather than the calculations.

MODULE COMPONENTS

Purpose: To describe the flow and component parts of the modules.


Seats at the Table

Mario wants to know how many people he can seat around a row of trapezoid shaped tables. He started by drawing the sketches below. Then he made a chart.

1 table = 5 people

2 tables = 8 people

3 tables = 11 people

4 tables = 14 people

Help Mario by answering the following questions. Use what you have learned about sequences and the recursive rule.

1. Complete Mario's chart:

Number of tables in a row:	1	2	3	4	5	6	7	8
Number of people seated:	5							

Think of the Number of people seated' as its own numerical sequence.

2. Mario says that he can seat 42 people around 13 tables. Is he correct? _____

3. Without drawing the tables, how many people can sit around 13 tables? _____ How do you know?

6 | PATTERNS 1b

Adapted from MARS

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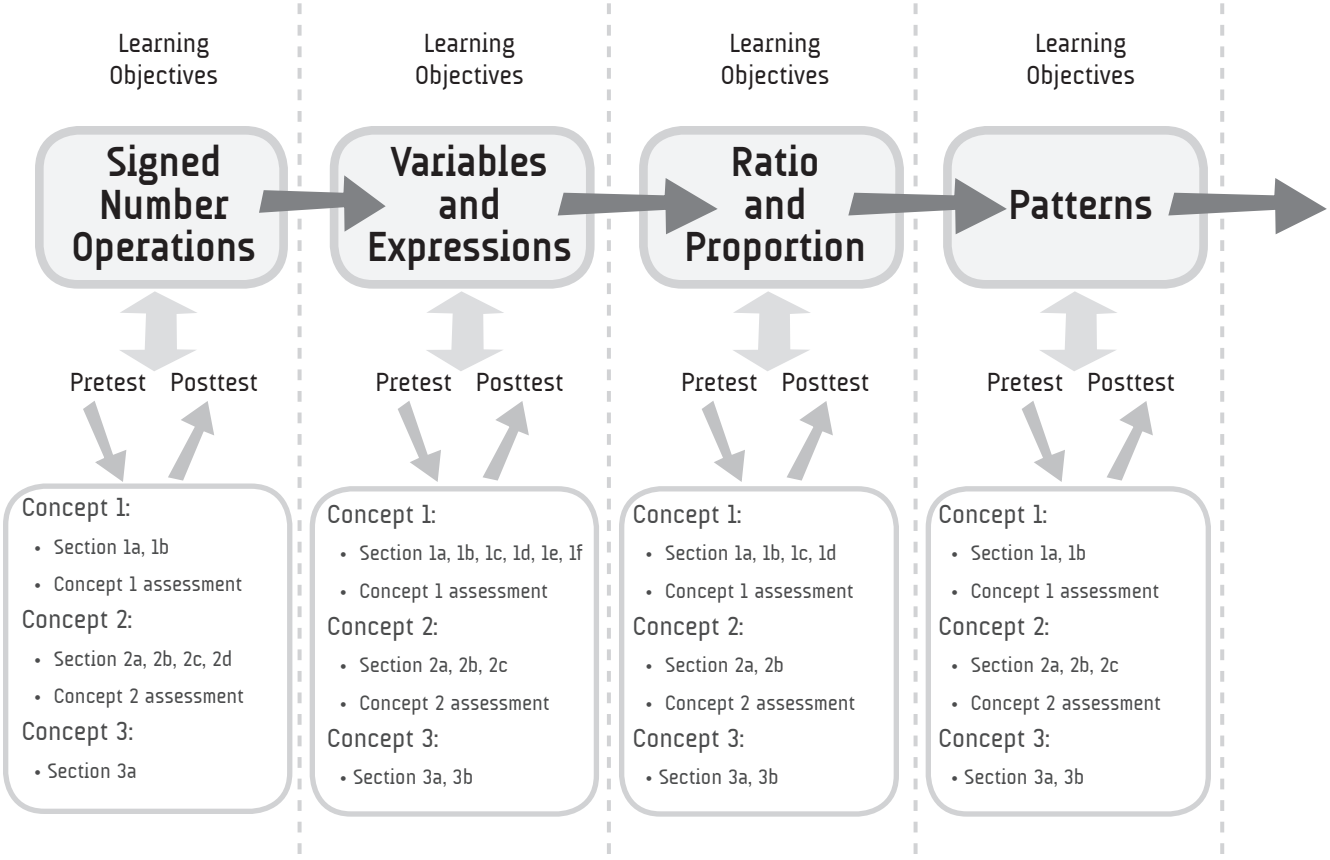
The partial graphic of the *Aim for Algebra* program shown below, represents the general structure of the modules.

The *Aim for Algebra* modules provide experiences in a targeted math content area. While there is a suggested order to the twelve modules based on the development and overlap of the content, module selection should be based on the needs of the students. Their needs might be served by one, two, or several of the modules, or they might need to experience the entire program.

All modules begin with a set of learning objectives that represent clusters of standards. These standards describe the math topics and tasks that students will explore in that module.

Each module is organized into three concepts. Each concept is separated into sections. The number of section in each concept will vary from module to module and from concept to concept. Since these sections represent a coherent math idea within the content area, the sections represent the optimum amount of materials to present to students at a given time.

Each module has a pre/post test and assessments at the end of Concept 1 and Concept 2. In addition, there is embedded assessment identified throughout each module.



THE *AIM FOR ALGEBRA* MODULES

Purpose: To provide a suggested order of the modules.

This suggested sequencing is based on the overlapping and scaffolding of mathematical concepts targeted in the program:

Signed Number Operations	Numbers Less Than Zero
	Models to Illustrate Signed Number Operations
	Applying Signed Numbers
Number Theory for Algebra	Language of Numbers
	Factors, Multiples, and Divisors
	Factoring Algebraic Expressions
Exponents	Exponents
	Properties of Exponents
	Applications of Exponents
Variables and Expressions	Variables
	Writing, Translating, and Evaluating Algebraic Expressions
	Simplifying Algebraic Expressions
Rational Numbers	Interpretations of Rational Numbers
	Operations with Rational Numbers
	Rational Expressions
Equations and Formulas	The Language of Equations
	Solving Equations
	Formulas
Ratios and Proportions	Ratios - Comparing Quantities
	Similarity and Scale
	Proportions
Patterns	Numerical Patterns and Sequences
	Patterns Describing Numeric and Geometric Relationships
	Connecting Geometric and Algebraic Patterns
Coordinate Plane	Parts of the Coordinate Plane
	The Equation of a Line: Graphing, Interpreting, Finding Slope
	Using the Coordinate Plane in Algebra
Proportional Reasoning	Introduction to Proportional Reasoning
	Application of Proportional Reasoning
	Unit Conversion and Dimensional Analysis
Inequalities	Basics of Inequalities
	Inequalities in One Variable
	Inequalities in Two Variables
Data and Probability	Collecting Data
	Interpreting Data
	Probability