

Oklahoma

Correlation between **MATH Connections** and the Oklahoma Priority Academic Student Skills (PASS) for mathematics in grades 9-12.

The short answer is that **MATH Connections** matches up extremely well with Oklahoma's PASS requirements, particularly considering that **MATH Connections** is a three-year core program and PASS covers four years. Oklahoma's PASS list for mathematics tracks very closely the NCTM Standards document, which guided the development of **MATH Connections** from the start. The following is a detailed comparison.

The headings and their descriptions are quoted directly from the Mathematics, Grades 9 - 12 section of Oklahoma's PASS document as it appeared on their website on 6/25/99.

PROCESS SKILLS

Preliminary Note: The Overview section of PASS for Mathematics says: "Though they are defined separately, the process skills should not be viewed as separate units of study. The mathematics curriculum should integrate some or all of the process skills into content-centered lessons." This is emphatically the case in **MATH Connections**. Problem solving, communication, reasoning, and connections are running themes throughout the entire three-year curriculum. For each of these parts and subparts, I have provided two things whenever possible:

- (1) general references to where this material is found in **MATH Connections**, and
- (2) some typical examples of how it is handled. (The typical examples are by no means exhaustive, of course. There are often many other instances for each item.)

I. Mathematics as Problem-Solving. The student will incorporate mathematical problem-solving strategies to solve problems from within and outside mathematics.

This is a major theme that recurs throughout all the **MATH Connections** books. Specific problem-solving strategies are emphasized by "Thinking Tips", a series of marginal notes that appear in appropriate places, particularly in the earlier books, to get students familiar with various aspects of the problem solving process. E.g.: Book 1a: pp. 13, 14; Book 1b: pp. 388, 389; Book 2b: pp. 297, 341.

The student will:

A. Apply problem-solving strategies to other disciplines and real-world situations.

Instances of this occur throughout every chapter of every book. Here are some particularly striking examples: Book 1a: Section 1.8, all of Chapter 4; Book 1b: Chapter 5; Book 2a: pp.110-119, 153-156, 175-177; Book 2b: Chapter 6; Book 3a: Section 4.1; Book 3b: Chapter 5

B. Identify the problem from a prescribed situation, determine the

necessary data and apply the appropriate problem-solving strategy.

This kind of thing occurs again and again in every book, particularly in the problem sets and explorations. Some of the many possible examples: Book 1a: pp. 26-30; Book 1b: pp. 387-398; Book 2a: Section 1.7, pp. 163-166, 180-183; Book 2b: pp. 311-322, 341-348; Book 3a: Section 1.1, Section 3.4, Sections 4.2-4.5; Book 3b: Section 6.1, Section 8.4

II. Mathematics as Communication. The student will use mathematical language and symbols to read and write mathematics and to converse with others.

This is actively fostered throughout all six books. In particular, the marginal devices "About Words" and "About Symbols" explain fundamental mathematical language and notation, and the "Discuss This" and "Write This" devices (signaled by the question-mark and pencil icons respectively) require students to talk and write about mathematics in every chapter.

The student will:

A. Demonstrate mathematical ideas orally and in writing.

See the "Discuss This" and "Write This" questions in every chapter, and also many open-ended questions in the Problem Sets.

B. Analyze mathematical definitions and discover generalizations through investigations.

Careful definitions of all mathematical terms used are clearly identified in the text. The most important ones are displayed as "Words (or Phrases) to Know"; the other defined terms are highlighted within the text when they are defined. Section 6.2 of Book 3b is devoted entirely to examining what it means to write a good, formal mathematical definition.

III. Mathematics as Reasoning. The student will use logical reasoning skills in mathematical contexts and real-world situations.

The habit of logical justification is begun in Chapter 1 of Book 1a and carried through the entire series. In the first-year books, the reasoning is largely (but not entirely) informal, asking students to justify their conclusions, etc. The second-year books show and ask for more formal justifications. Formal proofs in an axiomatic setting are presented and required in Book 3b.

The student will:

A. Prepare and evaluate suppositions and arguments.

This occurs throughout all chapters of all books. E.g., Book 1a, pp. 14, 46-47, 99-100, 173-174; Book 2a, Sections 2.2, 2.9; Book 3a, Section 1.1; Book 3b: Chapters 6 and 8.

B. Draw conclusions and identify counterexamples in mathematical context.

Throughout all books. The idea of a counterexample is introduced formally

at the beginning of Book 2a (see p. 38) and used routinely after that.

C. Justify mathematical statements through proofs.

Informal justifications are required from the beginning of the first book (see comments above). More formal arguments are introduced gradually in Books 2 (see pp. 151-152, for example). Formal proofs within axiomatic systems (not just in geometry) are examined and required in Chapters 6 and 8 of Book 3b. Mathematical induction appears in Section 7.2 of Book 3b.

IV. Mathematics as Connections. The student will appraise mathematics as an integrated whole and use mathematical concepts in other disciplines.

As its name implies, this is the defining theme of the entire **MATH Connections** program. See the following subheadings for specifics.

The student will:

A. Link mathematical ideas to the real world.

There are 100 different real-life applications within the Problems, Explorations, and situations of the texts. E.g., Book 1a: Sections 2.2, 4.1, 4.2; Book 1b: Section 6.1; Book 2a: pp. 134, 147-156; Book 3a: Sections 1.1, 3.1; Book 3b: Chapter 5.

B. Apply mathematical problem-solving skills in other curriculum areas.

The most common applications are to science (physics, chemistry, astronomy, biology, archaeology), but there are also applications to theater, music, business, and social sciences. E.g., Book 1a: pp. 49-51, 72-73, 214-215, 252-256; Book 1b: pp. 338-339, 397-398; 416-419; Book 2a: pp. 175-177, 187-189; Book 2b: pp. 354-356, 475-476; Book 3a: pp. 82-83, 99-100, 127-130, 150-151, 285-289; Book 3b: Sections 5.3, 5.5, 6.5.

C. Use mathematics in daily life.

Many instances of this occur throughout all books. E.g., Book 1a: Sections 2.2, 3.3, 4.2; Book 1b: Sections 5.3, 6.3, 7.5; Book 2a: pp. 110-118, 147-148, 175; Book 2b: 309-310, 384-389, 436-437; Book 3a: pp. 3-8, 93, 144-146, 205-208, 292-295; Book 3b: 309-313, 343-355, 549-550.

D. Relate one area of mathematics to another.

The integration of topics in **MATH Connections** is the blending of ideas from traditionally separate fields in ways that make it difficult (often impossible) to say where one begins and another leaves off. It presents mathematics as the subject is used, bringing ideas from a wide range of areas to bear on a question until the right combination leads to a satisfactory answer. In this sense, **MATH Connections** presents mathematics as a seamless fabric, perhaps with different patterns or colors in different areas, but with no clear boundary lines. E.g., Book 1a: The discussion of carbon dating in Section 4.4 uses first-degree equations (algebra), straight line graphs (geometry), least-squares differences

(algebra and statistics), and a graphing calculator (technology) to solve a problem in archaeology (natural science). All these tools come together naturally in this context. Book 2b: At the end of Section 5.4., algebra, geometry (the volume of a cone), and Cavalieri's Principle (usually in pre-calculus) are combined to calculate the volume of a sphere. Book 3b: In Chapter 6, properties of axiom systems (logic) are introduced by examining the rules of a simple card game and the probabilities of certain kinds of outcomes (Section 6.1). In Section 6.5, axiom systems are related to physics via the Law of the Lever and also to the arithmetic properties of the number systems. Such examples are not hard to find. In fact, it is difficult to find a section or major problem in **MATH Connections** which does not combine ideas from at least two traditionally separate mathematical topics or areas.

CONTENT SKILLS

For each of these parts and subparts, I have provided a listing of chapters and/or sections in which the skills are explicitly addressed. This listing is not exhaustive; the skill(s) in question may be used in other sections of the books, as well.

V. Algebra. The student will use algebraic concepts, symbols and skills to analyze, represent and solve a variety of problems.

Algebra is used in some significant way or another in every chapter of every book except the final chapter of Book 3b.

CORE SKILLS

The student will:

A. Communicate effectively using algebraic vocabulary.

Books 1: Sections 1.6 and 1.7, Chapters 2 - 6

Books 2: Sections 1.6, 1.9, 4.3, 4.4, 5.4, 5.6 - 5.8, Chapter 6

Books 3: Chapters 1, 2 and 5, Sections 6.6 - 6.8, Chapter 7

Appendix B (all books)

B. Differentiate between expressions, equations and inequalities and will perform the appropriate operation to evaluate or implement a solution.

Same as for part A.

C. Represent situations that involve variable quantities with expressions, equations, inequalities and matrices.

Same as for part A.

D. Use tables and graphs as tools to interpret expressions, equations and inequalities.

Books 1: Sections 1.6 and 1.7, Chapters 2 - 6

Books 2: Sections 4.3, 4.4, 5.6 - 5.8

Books 3: Chapters 1 and 2

Appendix B (all books)

E. Use calculators, computers or other technology to investigate and generalize algebraic concepts.

Appendices A, B, and C (all books)

Use of graphing calculators required throughout all books. Computer spreadsheets suggested in places (see esp. Appendix B).

F. Apply algebraic processes to become a creative mathematical problem solver in real-life situations.

Same as for part A.

G. Recognize and use the connections between algebra, other mathematics, and other disciplines.

Same as for part A.

H. Use the appropriate set of numbers to test the reasonableness of their conclusions.

Same as for part A.

I. Develop an understanding of the various number systems through investigation and analysis of their properties.

See especially

Books 1: Chapter 2

Books 2: Section 4.4

Books 3: Sections 6.6 - 6.8, Chapter 7

EXTENDED CORE SKILLS

The student will:

J. Demonstrate depth, breadth and sophistication in each of the algebra skills.

This, of course, is a matter of judgment. The spectrum of algebraic occurrences in **MATH Connections** is sufficiently rich and varied to allow students to develop depth, breadth, and sophistication.

VI. Geometry. The student will learn the fundamentals of geometry from several perspectives and select the appropriate form or forms to represent situations and solve problems.

The following chapters are primarily geometric:

Books 1: Chapter 3

Books 2: Chapters 1, 2, 4, 5

Books 3: Chapter 8

CORE SKILLS

The student will:

A. Use common geometric figures in problem solving situations by:

1. drawing and analyzing two- and three-dimensional figures;

Books 2: Chapters 1, 2, 4, 5

2. using properties of two- and three-dimensional figure to determine unknown values;

Books 1: Chapter 3

Books 2: Chapters 1, 2, 4, 5

3. determining and using relationships of congruence and similarity;

Books 2: Chapters 1, 2, 4

4. deducing properties and relationships of figures from given assumptions and information;

Books 2: Chapters 1, 2, 4, 5

Books 3: Chapter 8

5. applying geometric models in problem situations.

Books 1: Chapters 3, 4, 5

Books 2: Chapters 1, 2, 4, 5, Section 6.1

Books 3: Chapter 5

B. Use algebraic methods in coordinate and transformational geometry (reflections, rotations and translations) to:

1. translate between plane and coordinate geometry;

Books 1: Chapter 3

Books 2: Chapters 1, 2, 4, 5

2. deduce properties of figures;

Books 2: Chapters 1, 2, 4, 5

Books 3: Chapter 8

3. identify congruent and similar figures.

Books 2: Chapters 1, 2, 4, 5

Books 3: Chapter 8

EXTENDED CORE SKILLS

C. Incorporate vectors into the study of geometry by:

1. deducing properties of figures using vectors;

Books 2: extension to Section 3.4

2. using transformations, coordinates and vectors in problem-solving.

Books 1: Chapter 3

Books 2: Chapters 1, 2, 4, 5; extension to Section 3.4

D. Develop an understanding of the foundations (e.g., postulates, theorems) through investigation and comparison of various geometries.

Book 3b: Chapters 6 and 8

VII. Functions. The student will identify the important mathematical role functions perform and will use them to solve real-world problems.

Functions appear implicitly in Chapters 2 - 5 of Books 1. They are introduced formally in Chapter 6 of Book 1b and are used extensively after that in a variety of settings throughout all remaining books. Chapters focused explicitly on functions:

Books 1: Chapter 6

Books 3: Chapters 1, 2, 3

CORE SKILLS

The student will:

A. Recognize functions as an expression of relationships between different quantities by:

1. using tables, verbal rules, equations and graphs to represent and analyze relationships;

Books 1: Chapters 2 - 6; Appendices A and B

Books 2: Section 2.4, Chapter 3, Sections 4.1, 4.3, 5.1, 6.1

Books 3: Chapters 1, 2 and 3, Sections 4.8, 6.7, 7.8

2. interpreting information among tabular, symbolic and graphical representations of functions;

Same as for part A.1.

3. predicting the effects of parameter changes on the graphs of functions.

Books 1: Sections 3.4, 3.5, 6.4, 6.5

Books 2: Section 4.3

Books 3: Sections 1.5, 1.6, 2.3, 2.4, 3.4

B. Use functions to analyze real-world problems by:

1. describing phenomena with a variety of functions;

2. recognizing that a variety of problem situations can be modeled by the same type of function.

Same as for part A.1.

EXTENDED CORE SKILLS

C. Perform operations on classes of functions and describe their general properties and behavior.

Books 3: Section 6.7

VIII. Statistics. The student will use statistical methods to investigate, represent and analyze real-world problems. The student will:

CORE SKILLS

A. Sample, organize and interpret data, recognizing the role these play in making statistical claims.

B. Use various models to describe real-world data

Books 1: Chapters 1 and 4

Books 3: Sections 4.8 and 4.9

IX. Probability. The student will use probability to represent and solve problems.

CORE SKILLS

The student will:

A. Use experimental or theoretical probability, as appropriate, to represent and solve problems

B. Use simulations to estimate probabilities.

C. Generate and interpret probability distributions.

D. Interpret real-world applications of probability.

For all of the above, see

Books 1: Chapters 7 and 8

Books 3: Chapter 4, Section 6.1

X. Trigonometry. The student will demonstrate a variety of techniques and technology in applying trigonometry to solve mathematical and real-world problems.

Two full chapters are devoted explicitly to trigonometry: Chapter 3 of Book 2a and Chapter 3 of Book 3a. Trigonometric ideas and techniques are also used in other chapters.

CORE SKILLS

The student will:

A. Use trigonometric relations and functions to solve problems involving right triangles.

Books 2: Section 2.4, Chapter 3

B. Recognize the connections between trigonometry, geometry and

algebra.

Books 2: Chapter 3, Section 4.3

Books 3: Chapter 3

EXTENDED CORE SKILLS

C. Relate periodic phenomena to trigonometric and circular functions.

Books 2: Section 3.6

Books 3: Chapter 3

D. Understand the connection between trigonometric and circular functions.

Books 2: Sections 3.6, 4.3

Books 3: Chapter 3

E. Apply general graphing techniques to trigonometric functions.

Books 2: Sections 3.6, 4.3

Books 3: Chapter 3

F. Solve trigonometric equations and verify trigonometric identities.

Books 2: Chapter 3

G. Understand the connections between trigonometric functions and polar coordinates, complex numbers and series.

The groundwork for polar coordinates is laid in Section 4.3 of Book 2b. Complex numbers are described in an extension to Section 7.5 of Book 3b. Section 7.6 introduces infinite series, but not with respect to trigonometric functions.

XI. Calculus. The student will interpret the mathematics involving the study of change.

This area is not covered explicitly in **MATH Connections**, which is a three-year curriculum.

In anticipation of possible coverage in a fourth year, groundwork is laid in several places; see below.

CORE SKILLS

The student will:

A. Determine the maximum and minimum points of a graph and interpret the results in problem situations.

Books 3: Chapter 1

B. Recognize limiting processes by investigating infinite sequences and series and areas under curves.

Books 2: Sections 4.4, 5.3, 5.4

Books 3: Sections 4.8, Chapter 7

EXTENDED CORE SKILLS

C. Understand the conceptual foundations of limit, the area under a curve, the rate of change and the slope of a tangent line and their applications in other disciplines.

Books 3: Chapter 7

D. Analyze the graphs of polynomial, rational, radical and transcendental functions (e.g., trigonometric, logarithmic, exponential).

Books 3: Chapters 1, 2, 3